Essays on Asset Pricing and Portfolio Choice

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The Faculty of Economics, Business Administration and Information Technology of the University of Zurich hereby authorises the printing of this Doctoral Thesis, without thereby giving any opinion on the views contained therein.

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Part I

Introduction

Globalization of financial markets has increased dramatically over the last decades. Kose, Prasad, Rogoff, and Wei (2006) report that average gross stocks of foreign assets and liabilities across 71 countries have risen from around \$7 trillion in the early 1980s to \$76 trillion in the early 2000s. Global daily foreign exchange market turnover has increased from \$820 billion in 1992 to \$3,210 billion in 2007 (Bank for International Settlements (2007)). Besides deregulation, liberalization and financial innovation, technological advances were a key catalyst for this development. In particular, progress in computer and telecommunication technology have fundamentally changed the workings of financial markets. Financial institutions can gather, process and store as well as exchange information at much greater speed than before. Similarly, the cost of initiating, clearing and settling transactions and thus trading in general have been cut. Trade volume increased as a result of these improved conditions and the emergence of fully automated (algorithmic) trading.

These developments have fundamentally changed the behavior and complexity of financial markets. Financial economists are challenged to investigate markets that aggregate the interaction and decisions of millions of people across the globe. Fortunately, the last decades have also witnessed strong progress in macroeconomics and the very same advances in computer science have equipped researchers with increasingly powerful tools. Starting from groundbreaking work in the 1970s and 1980s, for example in Lucas (1982), economists were able to analyze dynamic models that capture key aspects of equity and bond returns (e.g. Bansal and Yaron (2004), Campbell and Cochrane (1999)) and business cycles (e.g. Smets and Wouters (2007)). Dynamic stochastic general equilibrium (DSGE) models have emerged as an important policy tool for central banks around the world. Despite considerable progress, numerous empirical observations demand further analysis. For instance, researchers have made only modest progress in understanding why carry trades, the strategy of borrowing in low interest currencies while investing in high interest currencies, have on average been profitable (forward premium puzzle) in the past 40 years.

This dissertation studies returns on financial markets, their connection to the real economy, as well as agents' portfolio choice. I consider the complex interplay between housing with other financial assets and the implication for investors' homeownership decision in different countries. Together with my coauthor I study a two-country economy to provide an explanation for the continued profitability of carry trades for ten different currency pairs. The distinct international focus in both studies reflects today's globalized world. We further study unemployment risk as a potential explanation for the large spread between equity returns and the low risk-free rate. All papers consider multiple agents interacting on national or international markets and the idiosyncratic risks they bear. The extension to multiple agents mitigates the problems surrounding the representative agent paradigm but naturally results in much more complex models. Due to complexity of the analyzed models, numerical dynamic programming is the common solution technique in all presented studies. I use state-of-the-art algorithms, computer programming as well as grid computing to solve and assess the power of these models.

The paper "Homeownership over the Life-Cycle: A Cross-Country Perspective" studies homeownership across three major countries – the US, Germany and the UK. Life-cycle ownership rates are hump shaped in all countries but quite different across countries, both in level and shape. Compared to Germans, US and UK households are generally more inclined to own homes and tend to purchase their homes earlier in life. The paper's objective is to identify cross-country differences that are likely to drive these empirical facts and to evaluate their validity in light of a life-cycle asset allocation model. Differences in life-cycle income patterns and housing credit market conditions explain the shape of early-life homeownership in the US and UK. Further differentiating households by family size and assuming that larger families have a stronger preference for ownership, the model replicates the stylized fact of hump shaped ownership profiles in all countries. The calibrated model closely predicts the empirically observed ownership profile in the US and UK, while the flat German homeownership profile demands further study.

The observation that high interest currencies tend to appreciate (e.g. carry trades work) is one of the main puzzles in international finance. Recent evidence suggests that the puzzle was most pronounced during the 1970s (Lothian and Wu (2011)), but, despite high financial integration, is still present today (Baillie (2011), Figure 1). The paper "International Diversification and the Forward Premium" provides a theoretical explanation based on investors' hedging desires. Within a two-country exchange economy with endogenous consumption, we show that habit preferences can lead to portfolio adjustments that explain the observed relation between interest and exchange rates. Pairwise combinations of the countries Australia, Germany, Japan, United Kingdom and United States yield ten country pairs used to assess the predictive power of the model. Under a realistic calibration, we vary the unobservable habit parameters to study how well our model predicts the observed co-movements in exchange and interest rates. When we restrict a country's habit parameters to remain the same across country pairs, our model is able to predict all forward premium regression slope coefficients within two standard deviations.

Simultaneously inferring all countrys' habit calibrations is a difficult optimization problem. In the paper "Calibrating/Estimating Economic Models Using Parallel Computing", I summarize various techniques to estimate parameters in macroeconomic models and describe the actual calibration strategy we employed in "International Diversification and the Forward Premium." To assess the quality of our model we use a variation of the simulated method of moments. As the target function is highly nonlinear we employ a global optimization routine. Each step of the global optimizer is parallelized and evaluated on a grid computer. Given the close resemblance to the simulated method of moments, I consider this calibration scenario to be very general.

Despite strong financial integration and innovation, markets remain far from complete. Agents cannot fully insure against all possible outcomes in the future. Further, individual agents usually face much greater uncertainty about their future income than suggested by fluctuations in GDP. Consequently, undiversifiable idiosyncratic risk has been studied as a potential explanation for the equity premium puzzle. However, up to now, studies have largely focused on two-agent economies. Incomplete markets in connection with unemployment is the topic of the final paper "Asset Pricing with Idiosyncratic Risk: The Impact of Job Loss". To allow for a realistic unemployment rate within the model, the number of agents has to be increased far beyond the literature standard of two agents. The paper uses the Smolyak approximation algrorithm to analyze six heterogenous agents. We show that undiversifiable labor income risk in the form of unemployment risk simultaneously leads to a sizable equity premium and a relatively low risk free rate.

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Part II

Homeownership over the Life-Cycle: A Cross-Country Perspective

Homeownership over the Life-Cycle: A Cross-Country Perspective

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This paper develops a life-cycle asset allocation model to study the level and life-cycle patterns of homeownership. The model is calibrated and confronted with data from three major countries – USA, Germany and UK. Under heterogeneous income profiles and uncertainty within a country as well as varying housing and credit market conditions across countries, the model replicates some key aspects of cross-country life-cycle ownership. Preference heterogeneity calibrated to varying family size improves the model's prediction leading to a good fit in two out of three countries.

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II.1 Introduction

Homeownership rates vary substantially across countries. Differences can be found both in overall levels and in life-cycle patterns. Housing investment constitutes a large fraction of homeowners wealth. Consequently the tenure choice is closely linked to households' general portfolio decision. I develop a life-cycle asset allocation model with housing to study ownership rates in three major countries. I show that part of the ownership rate differences can be explained by heterogeneity in economic fundamentals. In particular, differences in income profiles and their impact on household wealth accumulation lead to varying ownership predictions. I also show that preference heterogeneity can further emphasize the predicted ownership rates' hump shape bringing the model close to the data.

Housing claims a special role in the investor's asset portfolio as it serves a dual purpose: Occupying a home generates utility, while home equity is an investment vehicle. Since housing consumption and investment cannot be separated, understanding the homeownership decision is complex. The majority of papers that consider housing in a life-cycle framework have the goal to further explore the impact of housing on the household's other portfolio holdings, most notably stocks (e.g. Housing, Yamashita, and Flavin (2002), Cocco (2004) as well as Yao and Zhang (2005a) and Yao and Zhang (2005b)). These papers find that housing crowds out stock holdings providing an explanation for low stock holdings found in the data. Rarely, the focus is on ownership rate predictions. Cocco (2004) abstracts completely from renting and builds his model conditional on owning. Yao and Zhang (2005a) study asset allocation in the presence of housing and Li and Yao (2007) analyze life-cycle effects of house prices. While not the main model focus, the latter two papers yield an ownership rate prediction. They both predict a hump shaped ownership rate (as empirically observed) but underpredict young households' ownership rates and overpredict homeownership in later life.

Building on Davis, Willen, and Kubler (2006) and Willen and Kubler (2006), households in my model have access to an extensive set of assets. While most other studies rely on two assets, a stock and a mortgage, households in this model additionally have access to two types of uncollateralized borrowing. This gives households more flexibility in their choice of wealth accumulation. The introduction of a minimum housing constraint (as in Cocco (2004)) produces low ownership participation for young households without relying on computationally costly transaction costs. While several other studies work with only one overall income profile per country (usually the US), I use different income profiles across education groups and countries. Income profiles have substantially different shapes across education groups and together with income uncertainty are known to impact wealth accumulation and portfolio choice. I further allow for heterogeneity in required down payments and minimum house sizes across countries.

To my knowledge, this is the first study of homeownership within a life-cycle framework across different countries. I calibrate the model to the US, Germany and UK. These three major countries have very different life-cycle patterns of homeownership (see figure II.2) as well as credit market conditions. German homeownership rates are relatively low, peaking around 55%, while ownership rates in the UK and US peak around 85% but have different shapes. Required down payments on mortgage loans are smallest for the UK and largest for Germany. House values of the least expensive homes turn out to be much higher for Germany than for the US and UK.

Simulation exercises indicate that the model in its base calibration captures many aspects of empirical homeownership profiles. The model has difficulties to account for the below 100% peak in homeownership and the reduction in home ownership among the elderly. Introducing preference heterogeneity brings the model closer to the data.

The paper proceeds as follows. Section two introduces the model economy and section three discusses the model calibration. Section four discusses the model results. Computational and data details are presented in the appendix.

II.2 The model economy

In this section, I describe the general setup of the model economy. The setup is close to Cocco, Gomes, and Maenhout (2005) and Davis, Willen, and Kubler (2006).

II.2.1 The representative household

Preferences The economy consists of a large number of households who enter the job market at age G, retire at age J and live up to age T. Ex-ante they are different only in terms of their educational attainment. The representative household derives utility from consuming a numeraire good, denoted C_t and housing services, denoted H_t. H_t reflects both the physical size of the dwelling as well as its quality. The within-period utility function is denoted $U(C_t, H_t)$.

Endowments Each period, the household receives an exogenous stream of labor income (Y_t). The income stream is stochastic and made up of three components. A determistic part and a persistent as well as transitory shock. The parameters of the income process depend on the household's education level. Income is a function of age (t), education (e), other individual characteristics ($Z_{i,t}$), the persistent shock and the transitory shock. After retirement households are assumed to receive a fixed fraction of their income in the year prior to retirement as a pension. Thus, log income (y_t) follows

$$y_t = f(t, e, Z_{i,t}) + \nu_{i,t} + \epsilon_{i,t}, \qquad t \leq J$$

and

$$y_t = \lambda f(J, e, Z_{i,t}), \quad t > J,$$

where $\epsilon_{i,t}$ is distributed $N(0, \sigma_\epsilon^2)$ and $\nu_{i,t}$ is given by

$$\nu_{i,t} = \theta \nu_{i,t-1} + u_{i,t},$$

where $u_{i,t}$ is distributed $N(0, \sigma_u^2)$

II.2.2 Financial assets

The investment opportunity set is constant. The household can choose from five assets. Three of these assets represent debt instruments that the household is only allowed to short. To ease notation, positive holdings of such assets will be defined as a short position so that a universal nonnegativity constraint can be enforced. The following menu of assets is available:

Housing (θ^H) Housing is both a consumption good and an asset and generates a deterministic (gross) return of R_H. While empirically house price volatility is non-negligible, both at the aggregate and the individual level, a stochastic house price requires the introduction of a second endogenous state variable. Also to avoid a second state variable, I abstract from transaction costs. While transaction costs have been shown to have an impact on the home ownership decision (see Li and Yao (2007)), I believe that other factors play a more important role in determining ownership over the life-cycle.

Mortgage (θ^M) Households who decide to become homeowners have access to external financing through a mortgage up to a certain down payment. The down payment is enforced each period so that an owner faces the collateral constraint

$$\theta_t^H \delta - \theta_t^M \geq 0. \tag{II.1}$$

The mortgage rate is deterministic and will be denoted R^M. There is no scheduled payment plan as in Li and Yao (2007). Households are free to choose their level of debt financing each period.

Equity (θ^E) Equity yields a stochastic (gross) return of $\tilde{R}^E$ and constitutes the only savings instrument besides housing.[1]

Uncollateralized borrowing at low rate (θ^{Ul}) There are two sources of uncollateralized debt. Up to some level (ζ), the household can borrow at a relatively low rate, i.e.

$$-\theta_t^{Ul} + \zeta \geq 0. \tag{II.2}$$

Uncollateralized borrowing at high rate (θ^{Uh}) Beyond the level ζ, the household is still able to obtain debt financing, however has to recourse to a loan with an unfavorable interest rate.

Denoting $\theta_t = \{\theta_t^H, \theta_t^M, \theta_t^E, \theta_t^{Ul}, \theta_t^{Uh}\}$ as the vector collecting all assets defined above, I impose the short sale constraint

$$\theta_t \geq 0. \tag{II.3}$$

[1] Previous versions of this paper additionally included a riskless zero-bond as a means of saving. Incorporating such an asset in the existing framework does not significantly change the results since agents either invest in housing or in stocks.

II.2.3 Evolution of wealth and budget constraints

On the expenditure side the household faces the budget constraint

$$C_t = Y_t + \Xi_t + \theta_t^M + -\theta_t^E - D_t^o\theta_t^H - (1 - D_t^o)\alpha\theta_t^H + \theta_t^{Ul} + \theta_t^{Uh}, \tag{II.4}$$

where Ξ_t is current period's cash attained, D_t^o is an indicator for current period's ownership decision and α denotes the rental rate. Wealth accumulates according to

$$\Xi_t = -\theta_{t-1}^M\tilde{R}_M + \theta_{t-1}^E\tilde{R}_E + D_{t-1}^o\theta_{t-1}^H R_H - \theta_{t-1}^{Ul} R_{Ul} - \theta_{t-1}^{Uh} R_{Uh}. \tag{II.5}$$

II.2.4 Renting vs. owning

Following Cocco (2004), households cannot buy houses smaller than ($\underline{H}$). Usually it is difficult to buy only a fraction of a house (e.g. one apartment or one room). Thus an important characteristic of housing is that it is *lumpy* which is reflected in the so called "minimum housing constraint." To summarize, households that rent may choose a continuous quantity of housing, while those who decide to buy are confined to dwellings above a minimum size

$$D_t^o(\theta_t^H - \underline{H}) \geq 0. \tag{II.6}$$

Beyond that minimum size, I assume owners can, just as renters, freely adjust their home size.

II.2.5 The household's problem

Neglecting a bequest motive, the household will optimally deplete all its resources at time T and given its initial wealth (Ξ_0), solves the optimization problem at time $t = 0$

$$\max_{\theta_t, D_t^o} \quad \mathbb{E}\sum_{t=1}^{T}\beta^t U(C_t, H_t),$$

subject to the budget constraint (eq. (II.4)), the evolution of wealth(eq. (II.5)), the collateral constraint(eq. (II.1)), the borrowing constraint(eq. (II.2)), a set of short sale constraints(eq. (II.3)) and the minimum housing constraint (eq. (II.6)).

II.3 Model calibration

This section lays out the general and country-specific calibration.

II.3.1 Common parameterizations

A household enters the economy at age 21 (t=1) and lives up to age 80 (T=60). Households retire at age 65 (J=45). Following several others (e.g. Kiyotaki, Michaelides, and Nikolov (2011)), I assume the following modified Cobb-Douglas utility function[2]

$$U(C_t, H_t) = \frac{1}{1-\gamma}(C_t^{1-\phi} H_t^{\phi})^{1-\gamma},$$

where

$$H_t = D_t^o \theta_t^H + (1 - D_t^o)(1 - \psi)\theta_t^H.$$

The utility function implies a utility discount for renting. That means, occupying the same size and quality of dwelling as a renter provides less utility than as an owner. The idea is that owners are freely able to renovate and change their homes independent from a landlord and thus receive higher utility.

Table II.1 summarizes common parameters in the base case. The annual discount factor (β) and the curvature parameter are set to 0.95 and 2.00 respectively. Housing preference (ϕ) is set to 0.20 consistent with the average proportion of household housing expenditure documented in the 2001 Consumer Expenditure Survey. In absence of inflation, the interest rates and returns have to be calibrated to their empirical counterparts after inflation. Empirical estimates, discussed for example in Hu (2005), suggest that the average annual rental rate in the US has been around 7% for the years of 1993 to 1997. Accounting for inflation, I set the rental rate to $\alpha = 4\%$. Rental rates vary a lot depending on the location and dwelling quality and are thus generally difficult to calibrate. Compared to the literature my estimate is at the lower end of the spectrum and implies a bonus for renting in the base case. I use the rental discount in the utility function to balance the tenure decision. I set the low rate cutoff (ζ) to current household income. That is, households can borrow up to their current income (Y_t) at the low rate (R_{UL}) and beyond their current income at the high rate (R_{UH}). I set the real risk free rate to 2% and the risk premium to 4%. The resulting annual stock return is 6%. I set equity volatility to the historical average on the S&P 500 of $\sigma_{\tilde{E}} = 0.16$ and real housing return to $\mathbb{E}[\tilde{R_H}] = 1.00$. Goetzmann and Spiegel (2000) approximate real housing returns in the period of 1980 to 1999 to lie between -1.00% and 3.46% and Leigh (1980) estimates annual depreciation rate of housing units for the US to lie between 0.36% and 1.36%. Thus, subtracting an estimate of depreciation and maintenance cost, a 0% return on housing seems plausible.

II.3.2 Country-specific parameterizations

In this study, I investigate three different countries, the United States (US), Germany (DE) and the United Kingdom (UK). I assume that countries differ in terms of required

[2]The assumption of Cobb-Douglas utility is attractive because it yields scale independence. The implication of an elasticity of intra-temporal substitution of one between housing and non-housing consumption is only partially in line with empirical estimates. For a discussion see Li and Yao (2007).

Table II.1: Base case common parameters for both countries.

Parameter		Value
Discount factor	(β)	0.95
Risk aversion	(γ)	2
Rental rate	(α)	0.04
Housing weight	(ϕ)	0.20
Low rate cutoff	(ζ)	Y_t
Mortgage rate	$(\mathbb{E}[\tilde{R}_M - 1])$	0.04
Uncol. low (UL) rate	$(R_{UL} - 1)$	0.06
Uncol. high (UH) rate	$(R_{UH} - 1)$	0.15
Housing return	(R_H)	1.00
Rental discount	(ψ)	varies
Average stock return	$(\mathbb{E}[\tilde{R}_E])$	1.06
Std. stock return	$(\sigma_{\tilde{E}})$	0.16

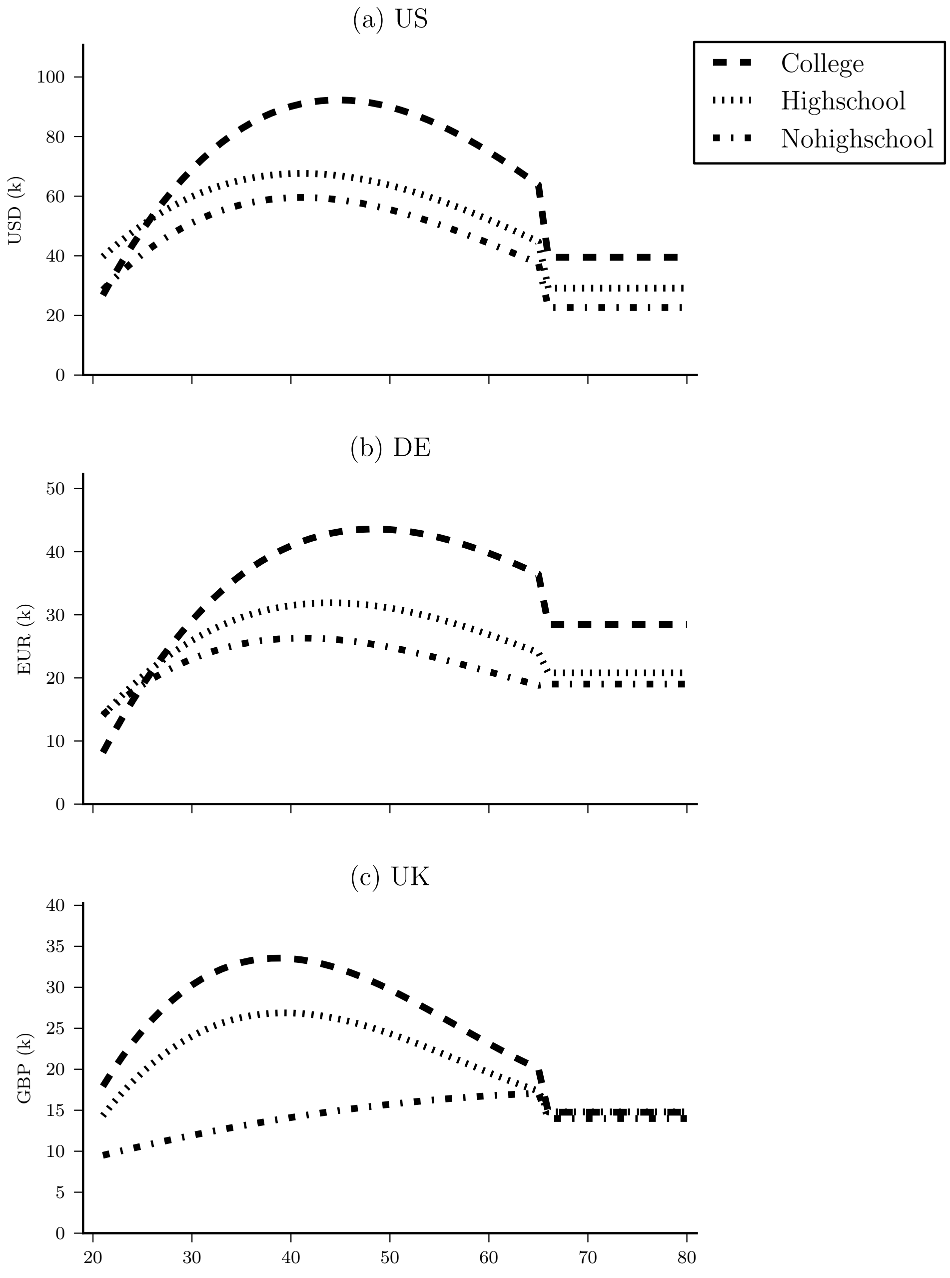
down payment, labor income profiles and the minimum house size potential owners face. I use country-specific panel data to obtain estimates for the necessary parameters. The country-specific panels are supplemented with data from the Cross National Equivalent File (CNEF). CNEF data attempts to harmonize the available panel data and allows for comparable cross-country studies. But because CNEF harmonizes a much broader set of countries, some variables that exist in the countries I study are not available in CNEF and I incorporate them manually from the country-specific panels.

Income profiles

Income profiles have been shown to heavily influence households' decisions on consumption (see Gourinchas and Parker (2002)) and portfolio choice (see for example Cocco (2004) and Yao and Zhang (2005a)) and are thus an important determinant in a life-cycle framework. Furthermore it has been widely documented (see Gourinchas and Parker (2002) and Attanasio, Series, Policy, and Bureau (1995)) that income profiles are substantially different across educational groups. To understand home ownership rates, it is crucial to take this heterogeneity into account. For this purpose three different income profiles are estimated for each country.

To make the estimated income profiles comparable to the literature, my estimation procedure closely follows Cocco, Gomes, and Maenhout (2005) who estimate income profiles and uncertainty for the US PSID data set. The estimation makes specific use of the panel dimension of my datasets. While Yao and Zhang (2005a) take the same approach, a large number of other studies uses the so called synthetic cohort methods developed in Deaton (1985) that essentially allow the construction of a panel from repeated cross sections. The first empirical paper to rely on this technique is Merz (1965). More recently Gourinchas and centering

Figure II.1: Income profiles



This figure displays income profiles for the US, Germany and the UK. I estimate one income profile for each education group within a country, differentiating between three educational levels: 1) college 2) high school 3) no high school. I run a fixed effects regression of income on a set of age dummies controlling for marital status and family size. The income profiles are constructed by fitting a fifth-order polynomial to the estimated age dummies. Income is broadly defined as pure labor income plus private transfers plus public transfers plus public pensions minus taxes in 2007 home currency.

Parker (2001) have made use of these techniques to estimate labor income profiles from the Consumer Expenditure Survey (CEX). Repeated cross-sections can generate much larger datasets but intrinsically compare different individuals over time. However, as time passes, the time dimension of the PSID and other popular panels grows in size, making the use of panel estimations increasingly attractive.

I estimate income profiles by regressing income on a set of age dummies controlling for family-fixed effects as well as marital status and family size[3] and finally approximating the resulting age dummies with a fifth-order polynomial. To maintain a large number of family-year observations an unbalanced fixed-effects model is estimated. I differentiate three education levels labeled "college", "high school" and "no high school." To control for education, the sample is split up by the household head's educational attainment. The variable used to split the sample (education with respect to high school) is provided as part of the CNEF data set. Following Cocco, Gomes, and Maenhout (2005) retirement income is approximated as the average income of households in retirement of age greater 65 divided by average income at age 65.

A broad definition of labor income is adopted. Government and private transfers bound a household's income from below. Thus pure labor income understates a household's true disposable income. I define labor income as pure labor income plus private transfers plus public transfers plus public pensions minus taxes in 2007 home currency, where each component represents the sum over all individuals in a household. It is important to exclude capital income from this definition since this part of household income is endogenous in the model. Unfortunately none of the panels differentiates between taxes paid on capital income versus taxes paid on other income sources. Thus it is impossible to directly back out the appropriate tax rate on *non-capital* income. The tax-computation software TAXSIM (developed by Daniel Feenberg) uses data on US tax law over the last decades and could be used to compute taxes on non-capital income for the US. Similar software, however, is not (publicly) available for the other countries. Thus I am forced to use a crude estimate for individual tax rates. Each year's tax rate is estimated from total taxes paid divided by total income.

The resulting income profiles are displayed in Figure II.1. Income profiles are quite heterogeneous across education groups but also across countries. The estimated income profiles for the US are very similar to those reported in Cocco, Gomes, and Maenhout (2005). Compared to Germany and the UK, US income profiles tend to be steep. Large amounts of life-time income are earned in the time between 40 and 50 while income is low in retirement. The UK income profiles for college and high school educated households peak around 40 and thus much earlier than German income (about age 50) and earlier than US income (around age 45). The replacement ratios for college educated households in the US, Germany and UK are 62%, 78%, 74% respectively.

Carroll and Samwick (1997) show that income uncertainty is an important determinant of life-cycle wealth accumulation. Since wealth accumulation and housing tenure are likely linked, the effects of income uncertainty on housing are also important. The stochastic part

[3]Regression results are reported in Table II.6.

Table II.2: Uncertainty estimates (%)

Country	Education	σ_u^2	σ_ϵ^2
US	College	1.57	5.21
	High school	1.15	5.93
	No high school	1.34	7.61
DE	College	1.38	2.89
	High school	1.23	3.53
	No high school	1.63	3.87
UK	College	1.47	6.91
	High school	1.16	6.14
	No high school	1.37	4.90

This table shows uncertainty estimates for the persistent (σ_u^2) and transitory (σ_ϵ^2) shock for the different countries. Estimates are computed following Carroll and Samwick (1997). First, I adjust income for economic growth, then I remove the predictable component using fixed-effects regressions and finally I run household specific OLS regressions of d-period income differences on a constant and a trend variable to obtain the parameter estimates displayed in the table.

of income is parametrized through the volatility of the persistent (σ_u) and transitory (σ_ϵ) shocks. I estimate these parameters using the algorithm proposed in Carroll and Samwick (1997). Differentiating income uncertainty into this permanent and transitory component is done in the following way. First income is adjusted for economic growth. Second, I remove the predictable component of individual income by obtaining the residuals of the fixed effects model used for the income profiles. Then I follow the appendix in Carroll and Samwick (1997) and run household specific OLS regressions of d-period income differences on a constant and a trend variable. The household average for the trend coefficient is the proxy for σ_u while the constant coefficient is the proxy for σ_ϵ. Table II.2 reports the estimates for the three countries. Estimates for the US are again similar to those found in the literature (e.g. Cocco, Gomes, and Maenhout (2005)). As expected, transitory shocks are more volatile the lower the household's education level for two out of three countries. The permanent shock is less volatile in all countries and education groups.

Several studies (Hubbard, Skinner, and Zeldes (1995), Carroll (1997) and Gourinchas and Parker (2002)) find the autocorrelation coefficient in the $\nu_{i,t}$ process to be close to one. In particular Hubbard, Skinner, and Zeldes (1995) find the AR(1) coefficient to be around 0.95. Thus, I choose $\theta = 0.95$ in the $\nu_{i,t}$ process.

II.3.3 Minimum house size

Supply of housing available for purchase is an important determinant households' ownership decision. To get an approximation of the minimum house size available for purchase, I rely on house values from the three panels. Table II.3 shows house value percentiles for the US,

Table II.3: House value (k) percentiles by country

Percentiles	US	DE	UK
1	8	30	27
5	27	80	37
10	46	100	44
25	82	150	58
50	129	200	84
75	201	300	136
90	328	380	216

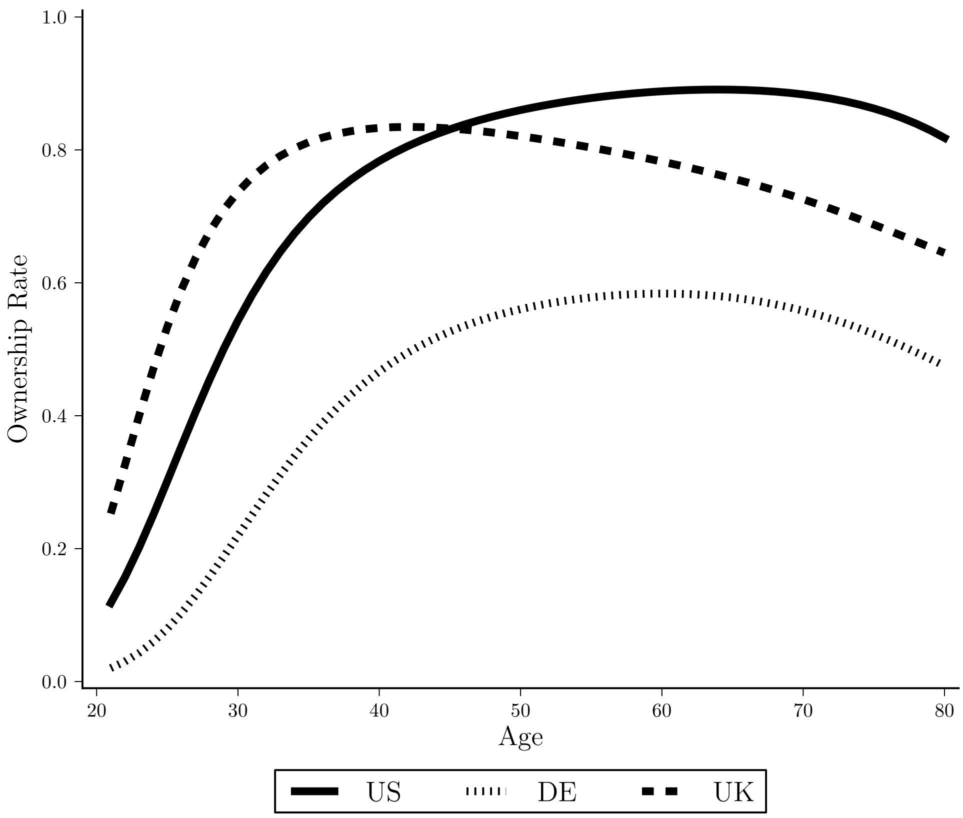
This table displays the distribution of house values as reported in the different panels across all households. The values are denominated in the country's currency and reported in thousands.

Germany and the UK. Since income profiles are in home currency, house values are also denoted in 2007 home currency and thus have to be converted with the according exchange rate to be comparable. To calibrate the minimum house value constraint I choose the 25th percentile of house values as the minimum house value within each country. Thus I set $\underline{H}^{US}$=\$46k, $\underline{H}^{DE}$=€150k and $\underline{H}^{UK}$=£58k. German housing is generally more expensive, at the same time, low-value housing in the UK tends to be slightly more expensive than in the US. While the choice of the 25th percentile is arbitrary, note that the relative house values are qualitatively the same for the other low percentiles. Interestingly the fact that owner-occupied housing in Germany (compared to life-time income) is generally more expensive than in the other two countries, already gives a first indication why German homeownership rates are rather low.

II.3.4 Down payment

Tsatsaronis and Zhu (2004) collect information on business practices and regulatory frameworks for mortgage finance across countries. On the basis of these characteristics countries are classified in three groups broadly homogenous with respect to structural features of their mortgage finance markets. Germany falls in the conservative first group, whereas the US is classified in the second and the UK is classified in the most "agressive" third group. According to Tsatsaronis and Zhu (2004) Germany's maximum loan to value (LTV) ratio is around 60% whereas the US maximum LTV is around 75-80% and the UK maximum LTV is 90-100%. I retain the general ordering in lending practices but for the analysis I set minimum down payments to $(1 - \delta^{GER}) = 0.25$, $(1 - \delta^{US}) = 0.15$ and $(1 - \delta^{UK}) = 0.10$ respectively.

Figure II.2: Empirical ownership rates



This figure displays empirical ownership rate profiles for the US, Germany and the UK. The empirical ownership profiles are estimated running a logit regression of ownership rate on a fifth-order age polynomial while controlling for marital status and family size.

II.4 Results

In this section, I describe how well the model can replicate economic data. Empirical ownership rates are constructed from the country-specific panel data by running a logit regression of ownership rate on an age polynomial controlling for marital status and family size. The resulting ownership profiles are reported in Figure II.2. The level of ownership rates exhibit substantial variation across countries. Average ownership rates in the US and UK are 74% and 76% respectively while the German ownership rate is only 47%. In addition, the life-cycle pattern of ownership rates is different across countries. The fastest increase in ownership rates is in the 20s for the US and UK but in the 30s for German households. UK ownership rates peak in the 30s while US and German ownership rates remain high up to the age of 60 before slightly decreasing too. Interestingly the early peak in ownership rates for the UK coincides with an early peak in the reported income profiles for college and high school educated households.

First, I evaluate the base case described in the calibration section. Then I study a

Table II.4: Household size across countries

HH Size	US	DE	UK
1	13.3	20.2	14.7
2	35.0	38.1	35.9
>= 3	51.7	41.7	49.5

This table displays the distribution of household sizes for the US, Germany and the UK.

model extension with heterogeneous preferences within the countries.

II.4.1 Base case ownership rates

Figure II.3 displays the simulated homeownership profile versus the empirical ownership profile for each country.

The model captures the early-life increases in US ownership rates well but is unable to simultaneously predict below 100% ownership rates in mid and late life. In particular the decrease in US ownership rates for retired households is not explained.

The model predicts a late entry into the housing market for German households (around the age of 30) followed by a fast increase of the ownership rate to 100%. In contrast German home ownership increase is slow but relatively steady. In principle the model replicates German low early-life ownership rates but lacks heterogeneity to match the slow increase in ownership rates. Different income profiles and uncertainty are insufficient to deliver this heterogeneity. For Germany the model predicts a drop in retired households' ownership rates.

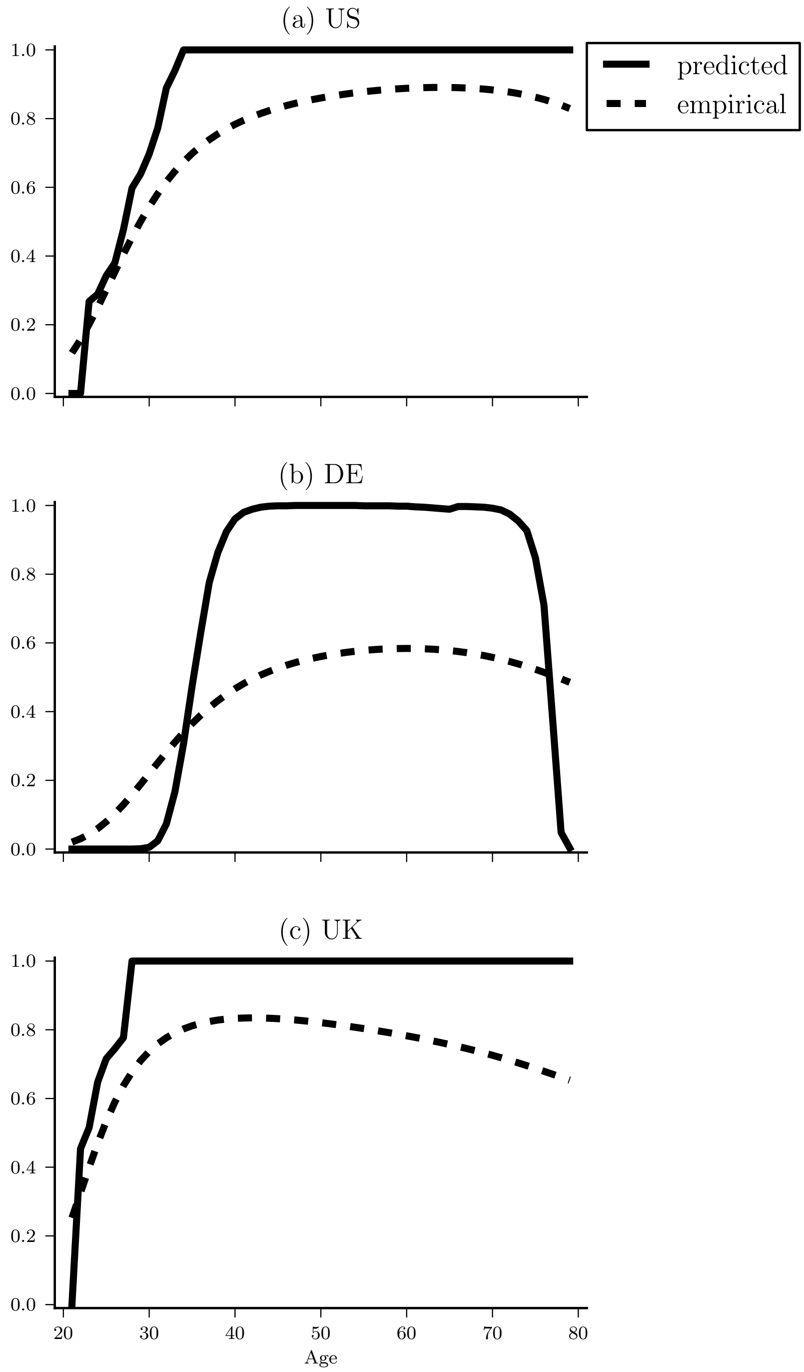
The fast increase in UK ownership rates is matched well by the model. Similar to the US case the model overpredicts ownership rates in mid-life and is unable to capture the reduction in ownership rates for retired households.

In general the model is able to either match early life ownership rate increases or the mid-life ownership level. In my calibration I opted for the first since it is more challenging and the model does quite well even across countries. If one were to match mid-life ownership rates, ownership rates increase much slower than the data counterpart. Low ownership rates among the elderly are hard to predict because of life-cycle wealth accumulation. Given the relatively low retirement income, agents build up large stocks of wealth during the working years. When agents are wealthy, financing considerations (mortgage rate vs. return/utility from housing) are secondary. In this case renting is only more attractive when the required (eq. (II.6)) ownership home ($\underline{H}$) is larger than the desired house size consumption.

II.4.2 Heterogenous preferences

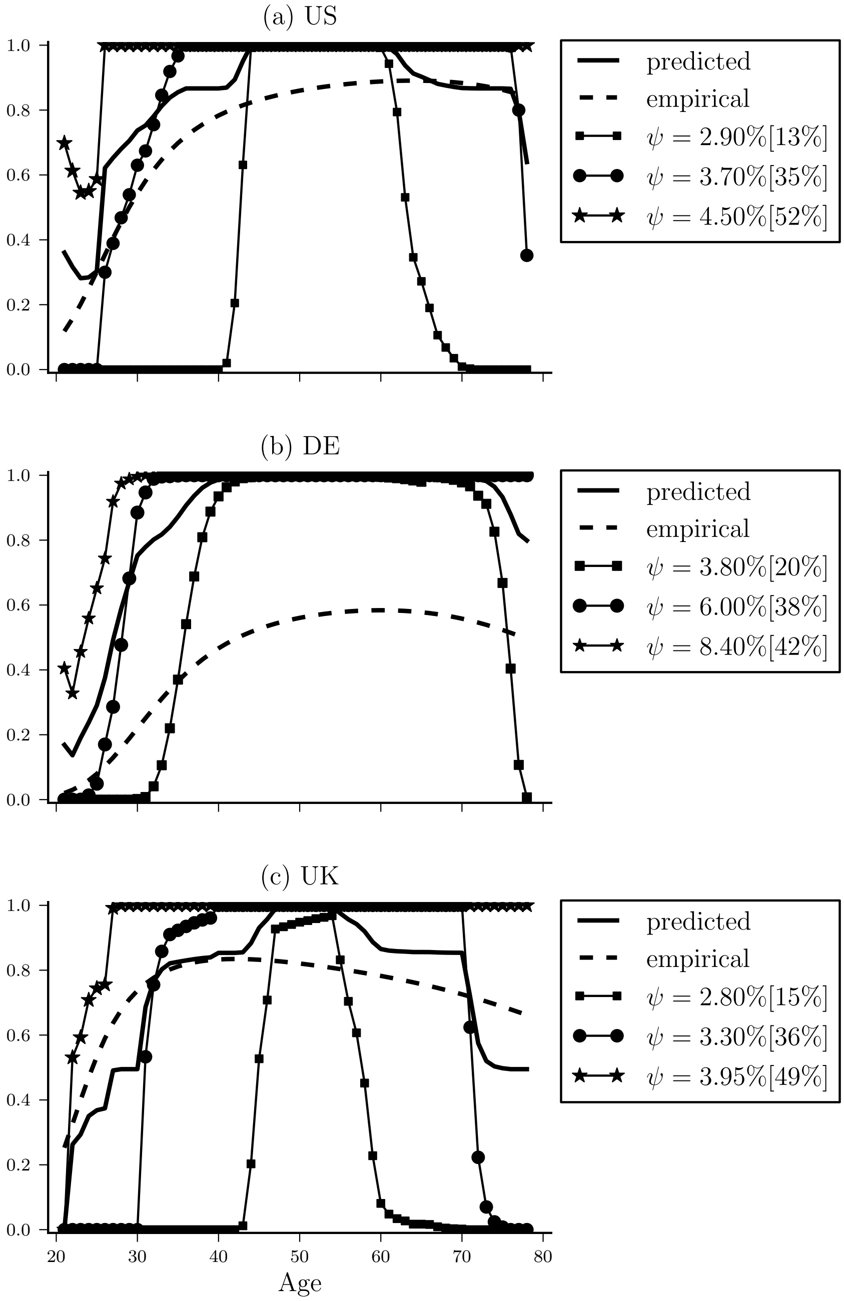
In this section I show that heterogeneous housing preferences (different ψ's) can resolve some of the model weaknesses outlined in the last section. The basic idea is to consider an

Figure II.3: Pred. vs. emp. ownership rates



This figure compares model predicted and empirically observed ownership profiles for the base case described in section II.3.1. In particular ψ is fixed at 3.90% for all countries in this calibration.

Figure II.4: Pred vs. emp ownership rates under heterogeneous preferences



This figure compares model *predicted* and *empirical* ownership profiles assuming heterogeneous housing preferences (different ψ's) within a country. From a grid of different types, I choose "interior preferences," that is ψ's for which households do not always rent or always own over the entire life-cycle. From this set I choose three equidistant rental discounts. Economies are then weighted according to the household size data in Table II.4. The first two types are weighted using the first two household size frequencies, the last agent receives the residual weight.

economy in which homeownership is made up of the tenure choice for agents with different housing preference. Figure II.4 displays results with heterogeneous agents for the three countries. Consider, for example, panel (a) which depicts predicted ownership rates under preference heterogeneity for the US. I assume there are three kinds of agents with ψ values of 2.90%, 3.70% and 4.50% respectively. The simulation result for an economy with only the low preference group is represented by a line with square bullets, with only the middle preference group with a circle bullet line and with only the high housing preference group with a star bullet line. Combining these three groups in one economy results in an economy with ownership rates represented by the line labeled "predicted." The figure shows that combinations of different housing preferences can lead to predictions that closely resemble real world data. In particular, the low ownership rate of the elderly can be explained by the low preference group exiting ownership, while part of the slow increase in ownership rates can be attributed to the low preference group entering the housing market late.

Whether the combined economy's ownership profile fits the data depends on the preference weighting. I believe one major driver of homeownership preferences is family size. Larger families usually settle down and have to plan long term. Making changes to the existing house might also be more valuable for families with children. Table II.4 reports household size frequencies for the three countries. German households are smaller than US and UK households. 58% of German households are small (households of size two or less) compared to 48% of US and 50% of UK households. Thus, German household size and the resulting low ownership preference can be another reason why German households tend to be renting.

To derive an economy-wide ownership profile, I have to specify the types of heterogeneous households I consider. I proceed as follows: First, I generate simulation results for a grid of housing preferences. Second, I identify those rental discounts (ψ) for which the simulated ownership profile is not always 0% or always 100%. Third, from these "interior" simulated economies, I choose an arbitrary number ($m = 3$) of equidistant economies which I then combine using weights generated from the family size data in Table II.4. The weighting is constructed as follows: The first $m - 1$ economies are weighted with the first $m - 1$ household size frequencies from Table II.4, while the last economy receives the residual weight. The choice of $m = 3$ appears sensible because single households, couple households and families are likely to have very different housing preferences.

Figure II.4 displays all ownership profiles obtained in this fashion. US and UK ownership profiles are now close to the data over the entire life-cycle. The German ownership profile still suffers from similar issues as in the previous section. Even the economy with low housing preference experiences 100% ownership in mid-life which stands in contrast to the below 60% maximum ownership rate in the data. The high weighting of the low preference economy brings the overall prediction closer to the data.

II.5 Conclusions

This paper studies life-cycle ownership rates across different countries. I use a life-cycle portfolio choice model to predict cross-country ownership rates. The combination of a minimum house size, different down payments and income processes results in acceptable predictions for early-life homeownership for two out of three countries. Predicting homeownership rates across the whole life-cycle requires the introduction of preference heterogeneity. I show that different preferences due to family size can lead to sensible predictions of life-cycle homeownership rates.

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Table II.5: Summary table for cross-country panel data

(a) US

	College				High School				No High School			
	mean	std	min	max	mean	std	min	max	mean	std	min	max
Income (k)	65.74	54.59	3.05	2370.00	44.44	23.51	3.02	491.29	33.57	19.93	3.07	223.06
Age	44.52	13.85	19.00	97.00	46.20	16.29	18.00	93.00	55.89	17.85	17.00	96.00
Marital St	0.82	0.39	0.00	1.00	0.83	0.38	0.00	1.00	0.82	0.38	0.00	1.00
Unem. Rate	6.46	1.37	4.20	9.70	6.58	1.40	4.20	9.70	6.83	1.46	4.20	9.70
HH size	2.91	1.35	1.00	11.00	2.88	1.34	1.00	12.00	2.78	1.39	1.00	16.00
Obs	36,054				32,395				19,437			

(b) DE

	College				High School				No High School			
	mean	std	min	max	mean	std	min	max	mean	std	min	max
Income (k)	41.93	22.63	3.34	276.31	29.50	15.06	3.07	375.92	25.16	12.93	3.11	203.70
Age	50.39	14.37	22.00	95.00	51.27	15.56	19.00	99.00	51.37	15.44	17.00	93.00
Marital St	0.72	0.45	0.00	1.00	0.72	0.45	0.00	1.00	0.74	0.44	0.00	1.00
Unem. Rate	10.19	1.62	7.20	13.00	10.12	1.62	7.20	13.00	10.03	1.61	7.20	13.00
HH size	2.56	1.25	1.00	8.00	2.48	1.18	1.00	13.00	2.66	1.39	1.00	17.00
Obs	17,332				55,641				17,041			

(c) UK

	College				High School				No High School			
	mean	std	min	max	mean	std	min	max	mean	std	min	max
Income (k)	25.99	14.59	3.00	152.59	21.20	11.47	3.05	213.16	15.86	8.29	3.04	87.79
Age	50.65	15.43	21.00	90.00	48.94	15.94	18.00	95.00	63.43	14.80	19.00	96.00
Marital St	0.88	0.33	0.00	1.00	0.88	0.33	0.00	1.00	0.74	0.44	0.00	1.00
Unem. Rate	7.25	1.86	5.10	10.40	7.42	1.87	5.10	10.40	7.55	1.86	5.10	10.40
HH size	3.06	1.26	1.00	8.00	3.11	1.25	1.00	9.00	2.40	1.29	1.00	10.00
Obs	3,367				7,539				7,402			

This table displays summary statistics for the PSID, SOEP and BHPS data. For detailed information on how I merged and harmonized the different data sets see section II.A.

II.A Detailed data description

My empirical analysis is based on PSID data for the US, BHPS data for the UK and SOEP (Wagner, Gert G., Frick, Joachim R., and Schupp (2007)) data for Germany. Partly, I can rely on synthesized data from Cross-National Equivalent File (CNEF) described in Burkhauser, Butrica, Daly, and Lillard (2000). CNEF attempts to facilitate cross-country studies by harmonizing information from different country-specific panels. However, working with CNEF and PSID allows me to see how the CNEF and PSID data sets relate, instead of simply relying on either one and at the same time gives me the flexibility to include all variables from the survey. Thus, whenever my data needs exceed the data provided by CNEF, I supplement my dataset with information obtained directly from the panels.

In the following I describe, how I set up the three panels in a systematic and symmetric way allowing me to perform the analysis undertaken in the main text. All three panels are complex in the sense that data is separated into different files that can be combined as needed using an identification key. The panels each have particularities, for example the way the data files are set up or individuals are tracked over time. In the following I describe the details of how I deal with these particularities and generate one homogeneous file for each panel. I use $ to denote the current year.

US/PSID Each year the PSID has a family-specific data set (fam$). I combine this data with the file "ind2005er" which contains all individuals ever surveyed as part of the PSID up to 2005. Next, I add household head information from CNEF to this data set and proceed to exclude households with missing or unusable data. While the individual file carries all households, household dropouts are not in the family file. I drop all households entering from the individual file and have not participated in the current year. Then I eliminate households depending on their response status, represented by the sequence number. A sequence number of zero indicates that the individual was either born or moved in after the current interview or did not respond for the current wave or had a mover-out non-response in the previous wave. Further, I drop households with sequence number 51 or above. These are individuals who live in institutions, moved out of the household or died before the current interview. After these adjustments, a few households present in the PSID but absent in CNEF are left (e.g. 23 households in the year 1980). I drop these households because I lack the CNEF information. When information for the current household head is missing, I check whether the data is available for the spouse and replace the spouse as head if possible. Afterwards I keep only household heads in the data set. Then I remove possible duplicate household heads by keeping only the male if there is a male and female household head otherwise arbitrarily keeping the first household head in the data set. In the PSID, the longitudinal weight is identical to the individual's cross-sectional sample weight for the most recent year of the longitudinal sample. Thus household weights are taken from the CNEF variable "w11102[final year]".

Germany/SOEP The starting point in constructing the German data set is the SOEP data file "PPFAD". It contains information on all households ever contacted as part of the SOEP survey with their respective unique individual identifier ("PERSNR"). For each year I merge this information on individuals into the CNEF data set for that year. Then I analyze the variable "$NETTO" which signals the interview status of the individual. If the household head is either labeled as child (value 20) or has no individual interview (value 30) or there is a gap interview for that year (value 31), I check whether the spouse can be used as a replacement for the household head, (that is the "$NETTO" value is not equal to 20, 30 or 31) otherwise the household is dropped. I further drop all individuals who are not the head of a household. Then I combine all year data sets into one large data set. The resulting data set is then augmented with the household weighting information from the file "hhrf." These weights some up to the total number of households in Germany. I use longitudinal household weights from the final survey year to weight the specific household.

UK/BHPS I obtain UK data from the BHPS files "indall" which carries information for all individuals surveyed in a given year as well as "hhsamp" and "hhresp" where the first carries technical household information such as the wave specific household identification number and the second carries information obtained during the interview process. I connect waves with the so called cross-wave personal identity number (PID). The household identification number and person identification number, wHID and wPNO respectively are newly assigned each wave and cannot be used to obtain longitudinal information. I start from CNEF data and then merge the file "xwaveid" which maps all individuals (PIDs) on wave-specific information such as wHID or wPNO. Finally I include year-specific information from "indall", "hhsamp" and "hhresp." I check whether interviews have been successful for head and/or spouse by checking the variable "ivfio" to be equal to -8 or greater or equal to 3. If I have no response for the household head, I attempt to use the spouse as head instead, finally retaining only household heads in the survey. I combine this individual data with the household data from "hhresp" and eliminate all heads that cannot be assigned a household in "hgen" (these are usually heads with "Individual questionnaire, no HH info"($netto = 19)) and households for which I find no corresponding household head (observation in "hresp" cannot be assigned to observation in main data file) as well as households with children heads (ivfio = 7).

Weighting households that drop out over time is tricky. For all panels I perform the following adjustment to the weights of a household that dropped out before the end of the sample. To account for changes in total cross-sectional weights, I use the weight in the last household response year relative to the sum of weights in that response year as the longitudinal weight for the drop-out household.

After these adjustments I have one data file for each country containing time series data on households with the same variables in each file. Now, I apply the same adjustments in all data sets. I subset data to lie in the period 1980 to 2005. To facilitate the analysis and to maintain comparability to Cocco, Gomes, and Maenhout (2005), I keep only households with male head. I drop household/year observations for which either the income measure or the age information is missing and keep only observations with positive weight. Further, I only include households with 1) income greater 3000 (home currency), 2) at least 3 consecutive time series observations and maximum increase (decrease) in income by 500% (80%). Especially the income uncertainty estimation is sensitive to outliers in the data. For this particular estimation I consider only households with at least 10 consecutive time series observations. Summary statistics for the resulting data files are reported in Table II.5. The results from regressing income on a set of age dummies and controls are displayed in Table II.6. The age dummies itself are all highly significant and omitted from the table.

Table II.6: Regression results

(a) US

	College		High School		No High School	
	coef	t-stat	coef	t-stat	coef	t-stat
Intercept	9.97	150.20	9.89	228.56	9.75	216.18
Household size	0.04	6.87	0.06	7.67	0.09	10.51
Marital status	0.37	17.79	0.25	11.39	0.20	6.93
T-bar	11.51		9.78		9.38	
σ_ϵ^2	0.32		0.29		0.30	
R^2-within	0.26		0.21		0.24	
F-stat	23.57		19.94		13.39	
n	3,133		3,313		2,073	

(b) DE

	College		High School		No High School	
	coef	t-stat	coef	t-stat	coef	t-stat
Intercept	10.11	150.70	9.54	230.06	9.37	135.50
Household size	0.07	4.95	0.11	14.82	0.13	9.81
Marital status	0.09	2.90	0.03	1.26	0.03	0.71
T-bar	8.46		8.71		8.69	
σ_ϵ^2	0.26		0.26		0.27	
R^2-within	0.19		0.15		0.15	
F-stat	8.72		16.32		5.62	
n	2,049		6,389		1,960	

(c) UK

	College		High School		No High School	
	coef	t-stat	coef	t-stat	coef	t-stat
Intercept	9.74	63.90	9.36	96.53	9.50	185.95
Household size	0.00	0.02	0.06	3.73	0.12	7.23
Marital status	0.14	2.19	0.05	1.01	0.10	2.02
T-bar	7.34		7.36		8.22	
σ_ϵ^2	0.30		0.29		0.28	
R^2-within	0.10		0.08		0.10	
F-stat	2.07		3.97		4.99	
n	459		1,025		900	

This table displays the results of fixed effects regression for the US, Germany and the UK. For each education level and country, log income is regressed on a set of age dummies, household size and marital status. The resulting age coefficients are omitted in this table. They are used to construct the income profiles in Figure II.1.

II.B Solution Technique

As no analytical solution exists, I use numerical methods to solve the model. First I describe how income normalization allows for state-space reduction and then I describe the implementation details of the numerical solution.

II.B.1 Income normalization

Given the recursive nature of the problem the inter-temporal problem can be rewritten as

$$\mathcal{V}_t(\Xi_t) = \max_{A_t} \quad \left\{u(C_t, \theta_t^H) + \beta\, \mathbb{E}_t[\mathcal{V}_{t+1}(\Xi_{t+1})]\right\},$$

where Ξ_t is the endogenous state variable (wealth) and $A_t = \{\theta_t, D_t^o\}$ is the vector of decision variables. D_t^o is a dummy variable with the value of one in the case of ownership.

Preference homotheticity allows the simplification of the household's optimization problem by exploiting the existing scale-independence. In particular, I normalize the problem by labor income (Y_t) so that the relevant state variable is then $\xi_t = \Xi_t/Y_t$ and denoting the vector of normalized asset holdings (i.e. $\frac{\theta_t^H}{Y_t}$, $\frac{\theta_t^M}{Y_t}$ and so on) as ϑ_t the decision variables are $a_t = \{\vartheta_t, D_t^o\}$. The household's budget constraint, eq. (II.4) becomes

$$c_t = 1 + \xi_t + \vartheta_t^M + \vartheta_t^{Ul} + \vartheta_t^{Uh} - \vartheta_t^E - D_t^o\vartheta_t^H - (1 - D_t^o)\alpha\vartheta_t^H - \vartheta_t^B, \tag{II.7}$$

where $c_t = C_t/Y_t$. The evolution of the normalized endogenous state variable is governed by

$$\xi_{t+1} = \frac{Y_t}{Y_{t+1}}(\vartheta_t^E\tilde{R}_E + D_t^o\vartheta_t^H R_H + \vartheta_t^B R_L - \vartheta_t^M\tilde{R}_M - \vartheta_t^{Ul}R_{Ul} - \vartheta_t^{Uh}R_{Uh}). \tag{II.8}$$

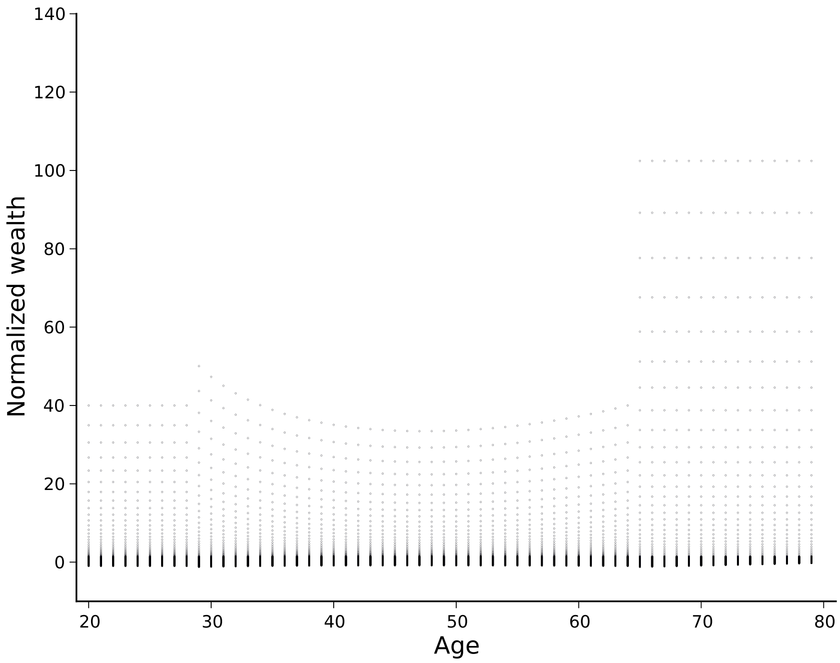
Denote normalized housing consumption $h_t = \frac{H_t}{Y_t}$, then the normalized value function $v_t(\xi_t) = \frac{\mathcal{V}_t(\Xi_t)}{Y_t^{1-\gamma}}$ can be written

$$v_t(\xi_t) = \max_{a_t} \quad \left\{\frac{1}{1-\gamma}(c_t^{1-\phi}h_t^{\phi})^{1-\gamma} + \beta\, \mathbb{E}_t\left[v_{t+1}(\xi_{t+1})\left(\frac{Y_{t+1}}{Y_t}\right)^{1-\gamma}\right]\right\},$$

subject to eq. (II.7), eq. (II.8) as well as the following modified versions of the collateral and borrowing constraint plus the nonnegativity constraint where $\varsigma_t = \zeta/Y_t$:

$$\begin{aligned} \vartheta_t^H\delta - \vartheta_t^M &\geq 0, \\ -\vartheta_t^{Ul} + \varsigma_t &\geq 0, \\ \vartheta_t &\geq 0. \end{aligned}$$

Figure II.5: Wealth grid US college educated household



This figure displays the wealth grid for a US, college educated household. The wealth gird is chosen as follows: The lower and upper bound depend on current expected income growth. I place 75% of the total 120 grid points between the lower bound and a normalized wealth level of 1.5, spacing the 90 points equally. Finally, the remaining points are placed between a wealth level of 1.5 and the upper bound and spaced exponentially.

II.B.2 Numerical solution

In the numerical solution of the model, I rely on value function iteration instead of the more commonly used policy function iteration. While both the renter and the owner problem in the final period are smooth, convex problems, the discrete ownership decision yields kinks in the value function and thus jumps in the policy function. Using policy functions to store tomorrow's choice is thus infeasible.

Numerical solutions require a state space discretization: I compute a discrete Markov chain to approximate the joint distribution of the stochastic process of equity return and income. I use the implementation of Tauchen's method (Tauchen (1986)) proposed in Knotek, Terry, and II (2008). Tauchen's method uses quadrature to discretize the state space. The innovations to equity return, persistent and transitory income shock are each approximated by two discrete states yielding eight discrete states for numerical integration. I check that the processes computed by the algorithm match the first two moments for all three shocks and the persistence θ is between 0.9 and 1.0.

The wealth-income ratio ξ_t is discretized into a grid of 120 grid points. Figure II.5 displays the final grid I use. The construction of the grid is important to make sure the algorithm converges. I proceed as follows: First the grid lower and upper bound are chosen according to expected income growth. This makes sure that I do not attempt to solve the problem at wealth levels in which the household cannot maintain positive consumption. Then I choose to place 75% of the wealth points between the lower wealth bound and a normalized wealth level of 1.5. The remaining 25% of the points are exponentially spaced between wealth level 1.5 and the wealth upper bound for that particular age. While the majority of agents face wealth levels between -1 and 10 during the simulation, I need to be able to assign a policy to the very few agents who accumulate very large amounts of wealth. During the simulation I check that no agent's wealth lies outside the proposed grid. The exponential spacing is convenient because it approximates the distribution of wealth much better than linear spacing and is thus likely to lead to a more accurate model solution.

The finite horizon problem can be solved by backward induction. The final period value and policy can be calculated easily: In the absence of a bequest motive households will always rent in the last period. In fact the household cannot take on any debt and will trivially choose not to invest in any asset. The household's disposable resources are made up of the final period retirement income plus cash attained through asset holdings. Using equation (II.5) the budget constraint can be written as

$$\begin{aligned} &C_T + \alpha\theta_T^H = \\ &Y_{T-1}\left(\frac{Y_T}{Y_{T-1}}\left(\vartheta_{T-1}^E\tilde{R}_E + D_t^o\vartheta_{T-1}^H R_H + \vartheta_{T-1}^B R_L - \vartheta_{T-1}^M\tilde{R}_M - \vartheta_{T-1}^{Ul}R_{Ul} - \vartheta_{T-1}^{Uh}R_{Uh}\right)\right). \end{aligned} \tag{II.9}$$

Available resources in T are given by the right hand side of eq. (II.9), that is $W_T \equiv$ RHS(eq. (II.9)). The intra-temporal problem at age T has the solution $\{C_T, \theta_T^H\} =$

$\{W_T(1-\phi), W_T\frac{\phi}{\alpha}\}$. The value function at time $T-1$ is then

$$\mathcal{V}_{T-1}(\xi_{T-1}) =$$

$$\underset{a_{T-1}}{max} \quad \left\{ Y_{T-1}^{1-\gamma} \left(\frac{1}{1-\gamma} (c_{T-1}^{1-\phi} h_{T-1}^{\phi})^{1-\gamma} + \beta\, \mathbb{E}_t \left[\left(\left(\frac{C_T}{Y_{T-1}} \right)^{1-\phi} \left(\frac{H_T}{Y_{T-1}} \right)^{\phi} \right)^{1-\gamma} \right] \right) \right\},$$

where

$$\frac{C_T}{Y_{T-1}} = \phi \left[\frac{Y_T}{Y_{T-1}} + \vartheta_{T-1}^E \tilde{R}_E + D_t^o \vartheta_{T-1}^H R_H + \vartheta_{T-1}^B R_L - \vartheta_{T-1}^M \tilde{R}_M - \vartheta_{T-1}^{Ul} R_{Ul} - \vartheta_{T-1}^{Uh} R_{Uh} \right].$$

Iterating backwards from age T, I obtain the optimal policy and value performing two separate optimizations, one conditional on ownership and one conditional on renting. I choose the tenure with higher value as optimal policy. The optimization is done with the optimization software "IPOPT" (Interior Point Optimizer) described in Wächter and Biegler (2006). For several "test" parameter combinations, I check that the optimizer DONLP2 (Spellucci (1999)) yields the same result.

The selection of starting points is crucial for the convergence of the optimizer. My general approach is to choose asset holdings to equalize the distance of next period's wealth to its lower bound and the distance of today's consumption to zero. This general approach works very well for this problem. The precise selection of starting points depends on the household tenure:

For renters, first, stock holdings are chosen in the above way and bounded at 0.001 below. Then I find investments in the uncollateralized asset with low interest rate which are bounded by 0.999 above. Finally I choose the uncollateralized asset at the high rate. My initial guess for household rent expenditure is $\alpha\frac{1+\xi_t-\vartheta_t^E+\vartheta_t^H R_{UL}+\vartheta_t^H R_{UH}}{\theta}$.

For owners, first I choose housing investments to be $\theta_H^{initial} = \underline{H} + 0.001$. Then uncollateralized borrowing is selected as in the renter case. If my asset choice implies next period's wealth above the wealth grid's upper bound, I simultaneously scale down stock holdings and home size by 1% increments. This choice of starting points leads to convergence on the whole grid for all computed parameter combinations in the state space.

Again referring to Figure II.5, I do the dual optimization for all nodes of the final age. I fill the gaps between neighboring nodes with tension splines. In particular, I use "TSPACK" (Tension Spline and Curve Fitting Package) described in Renka (1993) to obtain the continuous approximation of today's value and policy functions. Whenever the solver attempts to evaluate the value function outside of the grid (below or above), I use a first-order Taylor expansion to extrapolate the true value function. Equipped with this approximation of the value function, I proceed to compute the value and optimal policy at the previous age and then iterating backwards until age $t = 1$.

After computing solutions at all nodes, I use the optimal policies to simulate economies of size $10,000$ for each education level weighting the resulting economies with the number of households in the specific education group as reported in the country's panel.

Part III

International Diversification and the Forward Premium

Time-Varying International Diversification and the Forward Premium

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A version of this paper has been submitted to the Journal of International Money and Finance.

Abstract

This paper reproduces the slope of the uncovered interest rate parity (UIP) regression for ten country pairs within one standard deviation under rational expectations. We propose an infinite horizon dynamic stochastic general equilibrium model with incomplete markets. Heterogeneous investors experience varying risk aversion as a result of habit formation.

The underlying mechanism of the model relies on varying international diversification in the investors' portfolio choice decision. In response to their changing habit levels, investors' hedging desire varies over time. This leads to adjustments in interest rates. The habit-induced investment decisions are negatively correlated with movements in the exchange rate. This results in a negative correlation between interest rates and expected exchange rates, as implied by a negative UIP slope.

Depending on the magnitude of habits, the model is capable of reproducing positive as well as negative UIP slopes, as seen empirically in the data.

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III.1 Introduction

A large body of empirical literature[1] finds that high interest currencies tend to appreciate. This is surprising, since it implies that investors in high yield currencies benefit twice, once from the interest rate spread and once from the expected appreciation. Standard economic models predict exactly the opposite, namely that the uncovered interest rate parity (UIP) holds: high interest currencies should depreciate. The empirical phenomenon, usually referred to as the forward premium anomaly, is one of the most prevalent puzzles in international finance and has also given rise to the great popularity of carry trades[2].

Given the complexity and resilience of the puzzle[3], financial economists have been searching for a potential explanation ever since its discovery. Approaches toward a theoretical explanation emerge from three major directions: irrational expectations, market frictions or rational risk premia. This paper develops a two-country model under rational expectations without market frictions, attributing the forward premium to time-varying risk premia.

We assume that consumers form habits according to their consumption history. This changes the price of risk over time. When consumption drops close to the habit level, marginal utility increases and implied risk aversion rises. Contrarily, a large wedge between consumption and habit implies small risk aversion. Without habit, expected exchange rate (FX) appreciations always translate into a falling interest differential (confirm UIP). The introduction of habit induces shifts in investors' international diversification: Investors purchase foreign assets to hedge their consumption risk. The desire to hedge varies with different levels of income. Therefore, interest rate differentials carry time-varying risk premia for consumption growth. These risk premia are negatively correlated with FX returns. Thus, for sufficiently high habit levels, expected exchange rate appreciations can lead to increasing interest rate differentials (contradict UIP), as seen in the data.

The model's exchange rate is the ratio between tradable good prices in the two countries. We therefore assume Purchasing Power Parity holds for the tradable part of agents' income. This allows the model to generate realistic levels of inflation and FX returns simultaneously.

Markets are assumed to be incomplete on the international level. There is no asset that directly enables the representative investors to insure their income risk. This assumption is necessary to prevent countries from completely aggregating their individual risk, i.e. consume a constant percentage of the global income in tradable goods. The emerging country-specific consumption uncertainty impacts risk premia: they become larger and more varying, across time as well as across countries.

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[1] The discovery is attributed to Hansen and Hodrick (1980) and Fama (1984). For surveys see Hodrick (1987) and Engel (1996).

[2] Carry trade refers to the strategy of borrowing in low interest currencies while investing in high interest currencies.

[3] For a survey see Engel (1996). Important theoretical contributions include: Alvarez, Atkeson, and Kehoe (2009), Bacchetta and van Wincoop (2010), Bansal and Shaliastovich (2009), Bekaert (1996), Colacito (2006), Farhi and Gabaix (2008), Verdelhan (2010) and most recently Heyerdahl-Larsen (2012).

With habit levels common in the literature[4], we are able to reproduce the forward premium anomaly for ten different country pairs, composed of the five countries Australia, Germany, Japan, United Kingdom and United States. For eight out of those ten countries, the match is almost perfect, and for the remaining two the model remains within one standard deviation of the empirical observation.

This paper is related to the work of Verdelhan (2010). Verdelhan provides an explanation to the forward premium in the Campbell and Cochrane (1999) habit framework. He combines pro-cyclical interest rates with habit driven counter-cyclical risk aversion to replicate the anomaly. One restriction of his approach is that consumption has to be exogenous. In an international model, this implies the absence of trade, which Verdelhan achieves by assuming sufficiently large transportation costs. In the appendix, Verdelhan takes a first step toward a more diversified model, by reducing transportation cost and solving the planner's problem for the two countries. This paper takes the next step, by abandoning the planer and solving for a competitive equilibrium.

Thus, similarly to Verdelhan, we attribute the forward premium to rational risk premia, which vary over time due to habit formation. In our model, however, consumption is endogenous. We therefore allow for trade and international investment decisions. Theoretically, this allows for feedback effects between the two countries and generally for richer dynamics within the model. Empirically, it allows to replicate more and different moments. Most notably, we account for the low correlation between consumption and FX returns, commonly referred to as the Backus and Smith (1993) puzzle. Backus observes a disconnect between consumption and real exchange rates. Since asset prices crucially depend on correlation, matching this low correlation makes it very challenging to generate large and fluctuating risk premia. To our knowledge, this is the first model to account for the Backus and Smith (1993) puzzle (although in its nominal version), while matching negative UIP slope coefficients in a rational expectation framework.

The rest of the paper is organized as follows. In section two, we present our model, followed by a description of our numerical solution method in section three. In section four we describe our calibration. Section five discusses our model results and section six concludes.

III.2 The model

III.2.1 General setup

Real economy

This model describes an exchange economy of two infinitely lived countries, in which each country is endowed with two types of nondurable consumption goods, one tradable, one nontradable. Each country is represented by one agent[5]. In each period, agents receive a

[4]Campbell and Cochrane (1999), Verdelhan (2010).

[5]We name them agent H and agent F and they reside in country 1 and country 2.

share ϕ of their endowments in the tradable good $y_{TG,t} = \phi y_t$ and a share $1-\phi$ in the nontradable good $y_{NG,t} = (1-\phi)y_t$. The agents consider the foreign tradable good as a perfect substitute for the domestic tradable good and possess Cobb-Douglas preferences over the two consumption goods,

$$u(c_{TG,t}, c_{NG,t}) = \frac{2}{1-\gamma}\left(c_{TG,t}^{\psi} c_{NG,t}^{1-\psi}\right)^{1-\gamma},$$

where γ refers to the risk aversion and ψ to the preference for tradables. For ease of notation we refer to the vector of consumption $c_t = (c_{TG,t}, c_{NG,t})$, whenever an explicit distinction between tradables and non-tradables is not necessary.

Financial economy

Each country has separate exogenous price levels, determining the relative value of the currency. We measure the price level in terms of the nominal price of the tradable good in each country.[6] In addition, we assume that goods and assets can only be traded in the currency of the home country. Furthermore, we assume that Purchasing Power Parity (PPP) holds for tradable goods, thus determining the nominal exchange rate as

$$S_t = \frac{p_{1,t}}{p_{2,t}},$$

where $p_{1,t}$ and $p_{2,t}$ refer to the price levels (e.g. prices of tradables) in the two countries.[7]

Each country issues a one-period bond with no possibility to default. Denoting prices and nominal holdings of bonds, issued by country i by $q_{i,t}$ and $B_{i,t}$ respectively, and introducing a superscript to identify the country that chooses the economic variable, the home country's nominal budget constraint can be written as

$$C_{TG,t}^{H} \leq W_t^H + Y_{TG,t}^H - q_{1,t}B_{1,t}^H - q_{2,t}B_{2,t}^H,$$

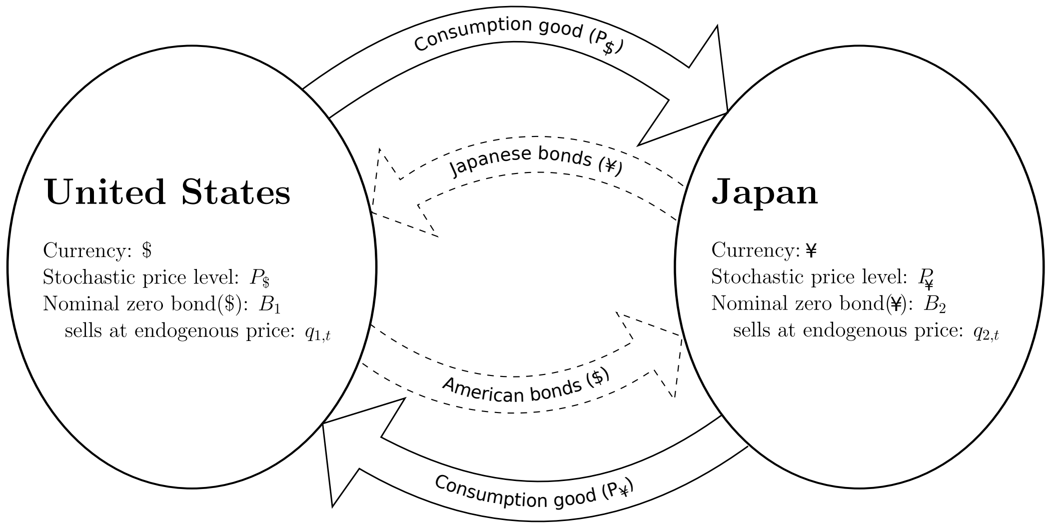
where $W_t^H = B_{1,t-1}^H + S_t B_{2,t-1}^H$ represents nominal wealth of country H. For this wealth, we assume a constant boundary on real debt

$$\frac{q_{1,t}B_{1,t} + S_t q_{2,t} B_{2,t}}{p_{1,t}} \geq \bar{w}^H.$$

[6]Prices of non-tradables have no impact on agents' decisions in our model.

[7]Given the empirical evidence on absolute and relative PPP it cannot be claimed that PPP holds for a general basket of goods. Burstein, Eichenbaum, and Rebelo (2005), however, show that PPP holds approximately for tradables, if one chooses the definition of tradable good appropriately. In particular, they distinguish between production and distribution of tradable goods. They argue that distribution is essentially nontradable. Based on this distinction they show empirically that even in times of extreme exchange rate fluctuations PPP holds approximately for tradables. Note that our model results are insensitive in the share of tradables in total income.

Figure III.1: Illustration of the market setup



This figure illustrates our model setup. Two countries trade consumption goods for nominal zero bonds, which are affected by stochastic price levels.

Uncertainty

Uncertainty enters the model through real and monetary shocks, where z_t denotes the vector of all such shocks. Shocks follow a first order Markov process with transition function $\Pi(z_{t+1}|z_t)$. Real shocks change the endowment of consumption good $(y_{NG,t}(z_t), y_{TG,t}(z_t))$ available to each country, whereas monetary shocks change the inflation rate in each country. This has two important implications. Firstly, monetary shocks determine the exchange rate through PPP. Secondly, although countries cannot default on their bonds, stochastic inflation implies a real consumption risk of holding bonds.

Note that the financial economy consists of only two bonds. Therefore, markets are generically incomplete.

Summary

Figure III.1 summarizes our model setup. Consider two countries, for example the United States and Japan. Each country has a stochastic income in its own good, a distinct currency and issues a nominally riskless zero bond. The United States sell some of their goods to Japan, while the Japanese issue bonds as a promise to repay in the future and vice versa. In equilibrium the net financial transactions will always equal the net real transactions.

Risk enters the model on the real side through stochastic income and on the financial side through stochastic inflation rates in each country, affecting the real payouts of the nominally secure bonds.

III.2.2 Habit utility

We assume investors value consumption only beyond their current habit level. The utility function, now supplemented by an external habit level[8], can be written as

$$u(c_t, h_t) = u(c_t - h_t) = u(c_{TG,t} - h_{TG,t}, c_{NG,t} - h_{NG,t}).$$

Following Constantinides (1990), Ferson and Constantinides (1991) and Heaton (1995) we specify investors' habit process[9] as a weighted average of past consumption, recursively written as

$$h_{t+1} = \rho h_t + \eta c_t. \tag{III.1}$$

For simplicity we consider the same habit level for tradables and nontradable goods, where we take nontradable consumption as a proxy for aggregate consumption in each country.[10]

This specification of habit increases the local curvature of the utility function, and thus, increases the risk aversion of agents. Moreover, risk aversion changes as agents experience different shocks to endowment. In times of consumption levels close to habit levels, marginal utilities are large and agents very risk averse. Contrarily, in times, when consumption is much higher than habit, marginal utilities are relatively small and the price of risk is low. Thus, habit formation allows for large, time varying risk premia.

Instead of calibrating the habit process parameters directly, we focus on the first two unconditional moments of the habit process, $\mathbb{E}[h]$ and $\mathbb{V}[h]$). They are more intuitive than the parameters of the habit process (IV.5). The original parameters are then given by

$$\rho = \frac{\frac{\mathbb{E}[h]^2}{\mathbb{E}[c]^2}\mathbb{V}[c] - \mathbb{V}[h]}{\frac{\mathbb{E}[h]^2}{\mathbb{E}[c]^2}\mathbb{V}[c] + \mathbb{V}[h]}, \tag{III.2}$$

$$\eta = (1-\rho)\frac{\mathbb{E}[h]}{\mathbb{E}[c]},$$

where $\mathbb{E}$ refers to the unconditional expectation and $\mathbb{V}$ to the unconditional variance.

[8]The literature distinguishes internal from external habit. We follow Abel (1990) in the use of external habit formation, commonly referred to as "catching up with the Joneses".

[9]Campbell and Cochrane (1999) and Verdelhan (2010) use a nonlinear, reverse engineered habit process, which has interesting implications in their framework. However, in our opinion, Constantinides's modelling of habit is the economically more intuitive choice.

[10]The good-specific habit levels are then formed as fractions of the aggregate habit level proportionally to the amount of tradables and nontradables in the economy.

III.2.3 Optimization problem

The optimization problem for each agent is

$$\max_{C_t, B_{1,t}, B_{2,t}} \quad \sum_{t=0}^{\infty} \delta^t u(c_t, h_t), \tag{III.3}$$

subject to the budget constraint, the law of motion of wealth and the borrowing constraint.

We seek a *competitive equilibrium*, that is a sequence of asset prices $q_t = (q_{1,t}, q_{2,t})$ and portfolio holdings $B_t = (B_{1,t}, B_{2,t})$[11], such that given q_t, the choice of B_t solves (III.3), subject to the agents' individual constraints and market clearing.

For each agent we can rewrite the sequence problem into the corresponding recursive problem. Define $z_t = (\pi_t^H, \pi_t^F, Y_t^H, Y_t^F)$, $\Psi_t = (W_t, z_t, h_t)$, then

$$V_t(\Psi_t) = \max_{C_t, B_{1,t}, B_{2,t}} u(c_t - h_t) + \delta\, \mathbb{E}_t[V_{t+1}(\Psi_{t+1})], \tag{III.4}$$

subject to

$$\begin{aligned}
C_{NG,t} &\leq Y_{NG,t}, \\
C_{TG,t} &\leq W_t + Y_{TG,t} - q_{1,t}B_{1,t} - q_{2,t}B_{2,t}, \\
W_{t+1} &= B_{1,t} + S_{t+1}B_{2,t}, \\
p_{1,t}\bar{w}^H &\leq q_{1,t}B_{1,t} + S_t q_{2,t} B_{2,t}
\end{aligned}$$

In addition, we impose the following market clearing conditions:

Bonds are in zero net supply

$$\begin{aligned}
B_{1,t}^H + B_{1,t}^F &= 0, \\
B_{2,t}^H + B_{2,t}^F &= 0.
\end{aligned}$$

Nontradable goods cannot be traded

$$\begin{aligned}
c_{NG,t}^H - y_{NG,t}^H &= 0, \\
c_{NG,t}^F - y_{NG,t}^F &= 0.
\end{aligned}$$

Aggregate consumption in tradables is equal to aggregate endowments

$$c_{TG,t}^H + c_{TG,t}^F - y_{TG,t}^H - y_{TG,t}^F = 0.$$

[11] Through the budget constraint, portfolio holdings imply a consumption path.

III.2.4 Model dynamics

Incomplete markets

The only financial assets in the model are the two bonds. As they fall short of spanning the state space, markets are incomplete. Completing markets would require to introduce assets, which allow to directly insure income risk. Such assets usually do not exist in the real world. In addition, the possibility to directly insure income risk would have undesirable model implications. In the first period, the agents would negotiate to fully share their income streams. I.e. each agent would receive a constant share of the global income in tradables.

Avoiding this unrealistic implication has three impacts on risk premia: they grow larger, more volatile and differ more strongly across countries. All these features are helpful in explaining the forward premium anomaly quantitatively.[12]

Reproducing the negative slope coefficient

Figure III.2 displays the underlying mechanism in our model. The objective is to reproduce the empirical observation that $E[\Delta s]$ is negatively correlated with $i^H - i^F$. Starting from an innovation in the income process, two major channels link exchange rates and interest rate differentials. The first channel we call the UIP effect. It is the effect found in a standard economic model compatible with a slope coefficient of one.[13] The second channel is novel to our model and depends on the time-varying risk aversion induced by investors' habit formation. Changing hedging needs have the potential to reduce the slope coefficient and even make it negative. Depending on the correlation between income growth and the exchange rate, both channels have slightly different dynamics. In the majority of countries income growth is correlated with FX appreications, but in some countries with depreciations. Therefore both cases are relevant.

Negative correlation

The middle part of Figure III.2 displays the case when income growth is correlated with an *appreciation* of the home currency ($E[\Delta s] \downarrow$)[14]. Clearly, investors anticipate a currency gain and will therefore demand a lower interest rate on home bonds ($i^H \downarrow$). This is the first channel or UIP effect. The second channel is novel and provides the explanation for the existence of the forward premium. In addition to the direct effect of a positive income shock on expected exchange rates, a positive income shock also induces more consumption increasing habit formation and thus risk aversion.[15] This stimulates the home country's

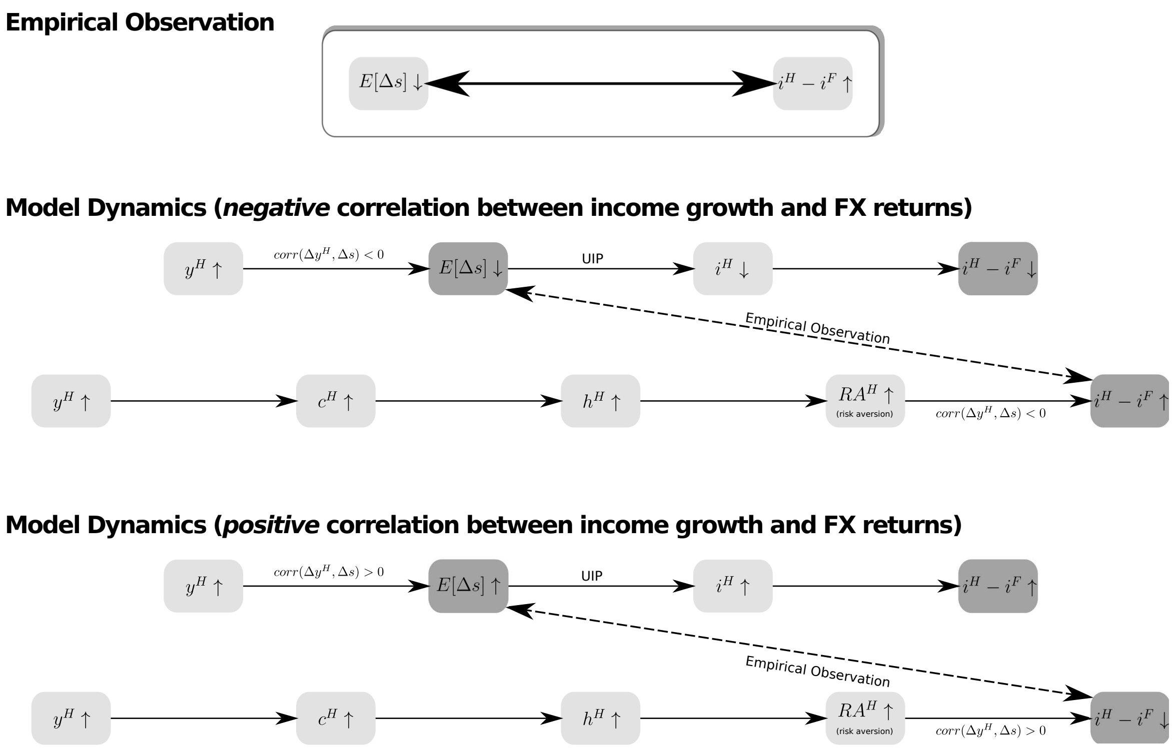
[12] See Engel (1996).

[13] See among many others: Fama and Farber (1979), Lucas (1982), Hodrick and Srivastava (1984), Hodrick and Srivastava (1986) and Engel (1992).

[14] Throughout the paper, we use the standard convention of denoting currencies as $\frac{HOME}{FOREIGN}$, therefore a decrease in the currency is equivalent to a appreciation.

[15] Risk aversion rises because habit reacts more strongly than expected consumption in our calibration. This effect is related to the question of how relative risk aversion reacts to changes in wealth. This has been

Figure III.2: Illustration of the central mechanism



This figure illustrates how the negative slope coefficient is reproduced in the model.
The upper panel states an equivalent formulation to a negative slope coefficient in the UIP regression. When the interest rate differential goes up, the expected exchange rate has to appreciate ($E[\Delta s]\downarrow$).
The middle panel shows how the puzzle can be resolved, when there is a *negative* correlation between exchange rates and income growth. The upper causality chain restates the standard UIP. The lower causality chain shows how this can be overcome by habit induced time-varying risk aversion.
The lower panel shows how the puzzle can be resolved, when there is a *positive* correlation between exchange rates and income growth. Again, the upper chain displays the UIP and the lower our habit induced risk aversion effect.

demand for foreign bonds, reduces the foreign interest rate and thus leads to a larger interest rate differential.

If the second effect quantitatively outweighs the first effect, this provides a possible explanation for the forward premium. The home currency appreciates at the same time as the interest rate differential increases.

Positive correlation

The bottom part of Figure III.2 displays the case when income growth is correlated with a *depreciation* of the home currency. Investors expect a depreciation of the home currency now. Therefore, they will demand a higher interest rate on home bonds to compensate for the expected decline in the purchasing power of their investment return. That is, according to standard theory, an FX depreciation is followed by an increasing interest differential. The mechanics of the second channel are almost identical to the negative correlation case. Higher income growth leads to habit formation and increasing risk aversion. Investors hedging desire rises. In contrast to the former case, now home assets provide a hedge against consumption risk. The interest differential falls as investors buy home bonds. Since both channels change signs, the negative slope coefficient can also be reproduced in the positive correlation case.

In summary, international diversification allows investors to hedge some of their income risk. As a result of income fluctuation and peoples' habit formation, the desire for international diversification fluctuates over time. The interest rate movements induced by this time-varying hedging desire has the potential to mitigate the UIP effect. Depending on the relative strength of both effects, the model can replicate a negative correlation between interest rate differential and expected exchange rates.

III.3 Computation

The dynamic programming problem (III.4) cannot be solved analytically. We therefore proceed to solve it computationally following methods in Judd (1998). To obtain a numerical solution the problem has to be discretized to a finite number of shocks. In practice this translates into approximating the estimated processes (i.e. income and exchange rate process for each country) by a discrete shock vector and an associated transition matrix. We simply follow the standard choice in the literature and use an implementation of Tauchen's algorithm (Tauchen (1986), Tauchen and Hussey (1991)).

We discretize the habit process into a discrete number of habit states. At the beginning of each period, habit is computed according to (IV.5). If the resulting value does not lie on the grid, we replace the computed value with the habit grid's closest node.

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an ongoing debate in the literature. However, recent evidence supports Arrow's original hypothesis (Arrow (1965) and Arrow (1970)) that relative risk aversion rises with higher wealth. See Halek and Eisenhauer (2001), Holt and Laury (2002) and Guiso and Paiella (2008).

Equipped with shock and transition matrices, the remaining relevant state space can be summarized by one endogenous state variable, net wealth of agent A. It summarizes the past actions of agent A. Wealth of agent B can simply be deduced through market clearing. Given the relevant state space of the economy, we use standard dynamic programming techniques to solve for the competitive equilibrium.

In particular we iterate over the agent's consumption policy. For the initial policy agents roll-over almost all of their debt, i.e. indebted agents pay back only a small amount of their loan in a two-period model. Then, in each step of the time iteration, we solve the nonlinear system of equations (see Appendix III.B.3, page 71) on a finite grid over net wealth and subsequently approximate the new consumption policy with cubic splines.

There is no theorem guaranteeing the convergence to or even the existence of a policy function satisfying the dynamic programming problem.[16] However, as long as we observe convergence toward a policy function, we know that it is a solution to the infinite horizon dynamic programming problem within the computational margin of error[17].

Finally, we simulate a large number of exogenous shocks for income and exchange rates and compute possible outcomes of the economy given the optimal policy functions. We perform the interest parity regression on the simulated data to test for the slope and observe additional implications of our model on various economic and financial variables.

III.4 Calibration

To assess our model's power to explain the forward premium anomaly, we calibrate the model to data for various countries. The set of countries, picked by historic economic significance, comprises Australia (AU), Germany (DE), Japan (JP), United Kingdom (UK) and the United States (US). The analysis puts special emphasis on the country pair United States and Japan, since these are the two largest economies, representing two dominant currencies; and most importantly as the anomaly is particularly robust for this country pair[18].

III.4.1 Data sources

For the calibration of our model, we need income growth, exchange rates, interest rates and trade shares. Except for trade shares, all data analysis is on the period from 1980 to 2010.

Income growth data is seasonally adjusted, in real terms and quarterly frequency and

[16] For a discussion see Duffie, Geanakoplos, Mas-Colell, and McLennan (1994) and Kubler and Schmedders (2005).

[17] The maximum deviations we allowed for were 10^{-10} for each individual FOC and 10^{-7} for the maximum change in consumption policies.

[18] Han (2004) performs a large cross-country, cross-period comparison to test whether the anomaly is universal. Performing regressions for varying time horizons in the range 1979 to 1998, he finds the percentage of observed negative beta coefficients to be 96% for the US and Japan.

provided by the Organisation for Economic Co-operation and Development (OECD), Eurostat and the Reserve Bank Australia.

Exchange rates are from Thomson Reuters Datastream and Eurostat, where we take the first day of each quarter in order to match quarterly income data. For the case of Germany we simply take the Euro as a proxy for Deutsche Mark.

As interest rates, we use 90 days Eurocurrency rates, again from Datastream. For Australia, there was no Eurocurrency rate available, thus we use "Interest rate on Bank accepted bills" as provided by the Reserve Bank Australia.

Finally, we obtain trade shares for the year 1999 from the World Trade Organisation, "Share of goods and commercial services in the total trade of selected regions and economies. "

III.4.2 Currency baskets

To calibrate the Markov chain, we need inflation and income data. While income data is readily available, tradable good inflation is not. Broad price indices, such as the Consumer Price Index (CPI), are not suitable since they incorporate both tradable and nontradable prices. More seriously, the usage of these indices would result in a model-implied exchange rate process that is completely different from the one observed in the data. This stands in sharp contrast to the paper's main goal of explaining the relationship between exchange rates and interest rates.

To avoid the above issues connected with price indices, we exploit the fact that the relation between tradable good prices in two countries is given as the exchange rate under PPP. More precisely, tradable good inflation in one country is measured as the valution of that country's currency against a broad index of other countrys' currencies. For each currency pair, we construct a currency basket of all remaining countries.[19] Tradable good inflation for one country is then derived as a weighted average of exchange rates of this country to all other countries in the basket. More formally, for a given country pair a, b, tradable good inflation is given as

$$\Pi_j = \sum_{\forall i \neq a,b} w_i S^{i,j} \qquad j = a, b,$$

where w_i is the weight of currency i in the basket and $S^{i,j}$ is the price of currency j in terms of currency i.

We choose the weights as the shares on world trade. More precisely, the relative value of the sum of each country's aggregate imports and exports with country i. As an example the currency basket for the country - pair US - JP is displayed in Table III.1. From here on, we will refer to the tradable good inflation process of a country simply as the country's (basket) exchange rate.

[19]It is convenient to exclude both countries in the basket in order to still obtain an exact match of the exchange rate when applying PPP.

Table III.1: Reference currency basket — Japan - US

Country	AU	CA	CH	DE	DK	FR	NL	NO	SE	SG	UK
Share	3.3	11.9	4.6	24.8	2.6	14.5	9.8	2.5	4.0	6.2	15.7

This table shows the composition of the reference currency basket for the country pair Japan - US. 1999 trade shares, obtained from WTO, are normalized such that they sum up to 100.

III.4.3 Estimation of exogeneous state variables

Equipped with the exchange rate process for each country, we can estimate the majority of the model parameters from data. In particular we estimate the exogenous shocks to income and inflation with a Vector Autoregressive Regression (VaR) of order one as

$$\begin{pmatrix} \Delta y_t \\ \Delta s_t \end{pmatrix} = \begin{pmatrix} \alpha_1 \\ \alpha_2 \end{pmatrix} + \begin{pmatrix} \theta_y & \theta_{y,s} \\ \theta_{s,y} & \theta_s \end{pmatrix} \begin{pmatrix} \Delta y_{t-1} \\ \Delta s_{t-1} \end{pmatrix} + \begin{pmatrix} \epsilon_{t,y} \\ \epsilon_{t,s} \end{pmatrix}, \tag{III.5}$$

where Δy and Δs refer to the change in logs of income and exchange rates against the currency basket, α and θ are the estimated coefficients and ϵ residuals.

Inflation of tradables — persistence

It turns out empirically that the coefficients $\theta_{y,s}$ and θ_s are universally insignificant. Thus, none of the analyzed countries show signs of significant persistence in nominal exchange rate returns. Table III.2 displays the persistence estimates and p-values for the ten different basket currencies analysed in our model. Only one currency comes close to the significance threshold. Therefore we simply set these values for all countries to zero. Arguably, this assumption makes a difference for our model economy. Without it, a second channel for a direct payoff effect opens up, working against the above proposed habit effect. However, in our opinion, the empirical evidence legitimates the assumption of zero exchange rate persistence.

Table III.2: Persistence of FX returns

Ctry1	Ctry2	Pers.	Pval
AU	DE	-0.01	0.89
AU	JP	0.07	0.38
AU	UK	-0.05	0.52
AU	US	0.07	0.38
DE	JP	0.11	0.20
DE	UK	0.09	0.34
DE	US	0.09	0.35
JP	UK	0.13	0.11
JP	US	0.06	0.49
UK	US	0.16	0.05

Persistence estimates and p-values of log returns on exchange rates over different currency pairs.

Markov chain approximation

The remaining results of the VaR regression (III.5) need to be discretized to accomodate our model. Therefore we discretize the process for each country into a Markov chain with 9 states. Table III.3 displays various statistics describing the result of the empirical estimation for the country pair US - JP, showing the high quality of the Markov

Table III.3: Markov chain approximation — Japan - US

Ctry	Parameter			Data	[s.e.]	Model
JP	FX returns	Mean	$\mathbb{E}[\Delta s]$	1.009	0.005	1.009
		Std.	$\sigma[\Delta s]$	0.055	0.004	0.044
	Income	Mean	$\mathbb{E}[\Delta y]$	1.005	0.001	1.005
		Std.	$\sigma[\Delta y]$	0.011	0.001	0.009
		Pers.	θ_y	0.233	0.093	0.166
		Corr.	$\rho_{\Delta s, \Delta y}$	-0.208	0.089	-0.207
US	FX returns	Mean	$\mathbb{E}[\Delta s]$	0.998	0.004	0.998
		Std.	$\sigma[\Delta s]$	0.045	0.003	0.035
	Income	Mean	$\mathbb{E}[\Delta y]$	1.007	0.001	1.007
		Std.	$\sigma[\Delta y]$	0.009	0.000	0.007
		Pers.	θ_y	0.350	0.074	0.254
		Corr.	$\rho_{\Delta s, \Delta y}$	-0.030	0.079	-0.030

This table compares the first moments of the Markov chain approximation for the two exogenous process FX returns and income growth to actual data.

chain approximation. There are some minor deviations in standard deviations and persistences due to the discretization, but correlation is matched precisely. Similar accuracy is achieved for all other country pairs.

III.4.4 Remaining parameters

Some parameters, especially preference parameters cannot easily be estimated from data. Table III.4 summarizes the remaining parameters. The parameters in the top panel are picked while the parameters in the lower panel are calibrated: They are chosen as to minimize the distance between the model simulated and the empirical forward premium

Table III.4: Calibrated parameters

Parameter (Quarterly)		Value
Share of tradables	ϕ	0.50
Discount factor	δ	0.99
Risk aversion	γ	2.00
Preference for tradables	ψ	0.50
Average habit level	$\mathbb{E}[h]$	0.93
Habit volatitility	$\sigma[h]$	0.0057

regression's slope coefficients.

The share of tradables in each country is set to 0.5[20], the discount factor to 0.99 on a quarterly horizon and finally the relative risk aversion to 2.00.

The habit parameterization is reported in the bottom panel. The average habit level is 0.93 and the habit volatility is 0.0057, roughly half the value of income volatility. This reflects the fact that habit is implicitly driven by changes in income, yet varies less than income.

III.5 Results

III.5.1 Simulation

Given agents' optimal policies, we simulate the model economy. The lower panel of Table III.5 displays the intercept and slope coefficient of a UIP regression using our model economy's data and corresponding actual data for the country pair US - JP. Our model matches both the slope and the intercept almost within one standard deviation. The theoretical values of the model are an approximation to the simulated value.[21]

The upper panel of Table III.5 shows the correlation between consumption growth and FX returns. It is known as the Backus and Smith (1993) puzzle (for real exchange rates), that these correlations are surprisingly low or even positive although standard economic theory would predict them to be close to -1.

Correlations are crucial in any explanation related to risk premia. Correlation directly affects covariances, which determine the stochastic discount factor and thus risk premia. Therefore, the capacity of our model to account for these low correlations is an important advantage over other risk-premia related explanations such as Bansal and Shaliastovich (2009) or Verdelhan (2010).

III.5.2 Impact of habit

Varying habit levels

Figure III.3 displays the impact of the local curvature, as implied by average risk aversion[22], on the slope coefficient for three different levels of habit volatility.[23] In the case of relatively small habit levels the model simply reproduces the uncovered interest parity, i.e. $\beta \approx 1$.

[20]Burstein, Eichenbaum, and Rebelo (2005) estimated the share of nontradables for ten different countries and found values between 0.31 and 0.57.

[21]For comparison and plotting purposes, it is inconvenient to use simulated slope coefficients, because of the introduced standard errors. Therefore, for the purpose of the regression, we make the assumption that ϵ_{t+1} is uncorrelated with time t expectations (The violation of this assumption is induced by the discretization of the state space). This allows us to compute approximate, yet exact slope coefficients, see Fama (1984).

[22]Local curvature is given by $\frac{\gamma}{c-h}$.

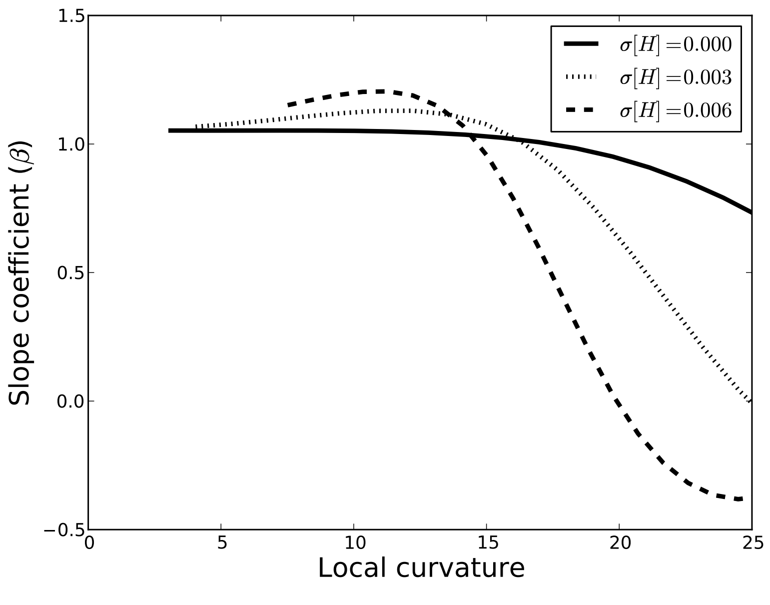
[23]The three curves have different starting values for local curvature, since combining high volatilities with low average habit levels results in negative habit persistences ρ (see (IV.6) on page 80).

Table III.5: Empirical versus model implied — Japan - US

Parameter (Quarterly)	Data	s.e.	Model (sim.)	s.e.	Model (th.)
$\rho^{JP}_{\Delta s,\Delta c}$	0.22	[0.08]	0.13	[0.01]	
$\rho^{US}_{\Delta s,\Delta c}$	−0.10	[0.08]	−0.06	[0.01]	
α_{UIP}	0.03	[0.01]	0.02	[0.01]	0.02
β_{UIP}	−0.63	[0.25]	−0.36	[0.15]	−0.38

The first panel compares the model implied correlation (ρ) between real consumption growth (Δc) and FX returns (Δs) with the data. Consumption growth and FX returns are on a quarterly basis. FX returns are denoted as home over foreign, so for Japan as $\frac{¥}{\$}$ and for US as $\frac{\$}{¥}$.
The second panel compares the results of the UIP regression. α_{UIP} refers to the intercept and β_{UIP} to the slope. We report two model values. The actual simulated value with standard errors and a theoretical approximation, which we use for plotting and calibration.

Figure III.3: UIP slope coefficient — Japan - US

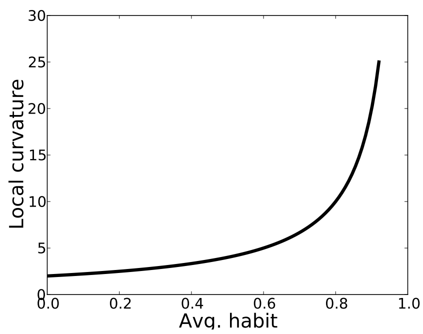


This figure displays the UIP slope for the country pair Japan - US over local curvature, as implied by average habit. The three lines represent different levels of habit volatility.

The UIP effect dominates because the habit level is too small to create large enough risk premia.

For a constant level of habit, i.e. zero habit volatility (the solid line), the UIP channel still dominates. Only at high local curvatures the habit channel starts to play a role and slowly reduces the slope coefficient. However, the coefficient remains close to one.[24] Note, that a constant habit level does not imply constant risk aversion. Since consumption varies, so does the spread $c-h$ and thus the local curvature.

Figure III.4: Local curvature



The two dotted lines in Figure 3 display cases of nonzero habit volatility. These correspond to parameterizations in which consumption shocks impact next period's habit level (i.e. $\nu \neq 0$). As average habit levels and thus risk aversion rise, the habit induced international diversification effect becomes increasingly important and finally outweighs the UIP effect. For a habit volatility of 0.003, high habit levels drive the slope coefficient down to 0. For a habit volatility of 0.006, the model predicts negative slope coefficients for average habit levels around 0.9 (i.e. an implied local curvature of 20, see Figure III.4).

III.5.3 Multiple countries

In addition to the detailed analysis for the country pair US - Japan, we apply our two-country model to nine other country pairs. These are formed by pairwise combination of Australia (AU), Germany (DE), Japan (JP), United Kingdom (UK) and the United States (US). Initially, we estimate a Markov chain for each country pair as described in the calibration section, then we solve for optimal policies and compute the model implied UIP slope coefficient. Finally, we compare these slopes to the data.

We explore three calibration scenarios. In the first scenario we pick a common habit calibration for both countries, i.e $\mathbb{E}[h_1] = \mathbb{E}[h_2]$ and $\sigma[h_1] = \sigma[h_2]$. The objective is to show that our model is in principle capable of explaining the observed forward premium for each country pair. In the second scenario, each country has its own habit parameterization, which is the same across country pairs. The idea is to infer country preferences and show that the model is able to explain the puzzle for all country pairs simultaneously. In the third calibration scenario we challenge the model with a habit process constant across all countries.

Table III.6: UIP slope coefficient — separate calibration

Ctry1	Ctry2	$\mathbb{E}[h]$	$\sigma[h]$	Model β	Emp. β	s.e.
AU	DE	0.85	0.0006	0.25	0.25	[0.21]
AU	JP	0.84	0.0074	0.12	0.10	[0.27]
AU	UK	0.81	0.0070	-0.19	-0.19	[0.23]
AU	US	0.95	0.0043	-0.04	-0.04	[0.17]
DE	JP	0.88	0.0054	0.12	0.12	[0.29]
DE	UK	0.82	0.0026	0.27	0.27	[0.18]
DE	US	0.96	0.0024	-0.04	-0.03	[0.21]
JP	UK	0.90	0.0068	-0.68	-1.05	[0.38]
JP	US	0.93	0.0057	-0.39	-0.63	[0.25]
UK	US	0.96	0.0029	-0.04	-0.04	[0.19]

This table reports the empirical vs. model implied slope coefficient for the calibration case: one habit parametrization per country pair. The first two columns refer to the country pair. The next two columns to the common habit preferences for each country pair. Finally, the last three columns compare the model implied value to the empirical observation.

Separate calibration — one habit parameterization per country pair

Table III.6 shows the result for a country pair specific habit calibration. Each country pair is analyzed separately. We assume the same habit parameterization for the two countries. The first two columns refer to the countries, the next two columns to the common habit parameterization. In the last three columns we compare the model implied slope coefficient with the empirically observed slope coefficient.

For the majority of country pairs the match is almost exact. Exceptions are Japan - US and Japan - UK. The model has difficulties reproducing these highly negative slope coefficients. Still the model implied β remains within one standard deviation for every country pair.

Simultaneous calibration — one habit parameterization per country

In this calibration, each country is assigned its own habit calibration. That is, we pick an average habit level and a habit volatility for each country to match the slope coefficients for all country pairs simultaneously. Each country has the same habit parameterization, independent of which country it is compared to. The results are reported in Table III.7. The first two columns refer to the country pairs. The next four columns display the habit parameterization for country 1 and country 2, respectively. Finally, the last three columns compare the model implied slope coefficient to the empirical observation.

Note that each country keeps its average habit and habit volatility across different country pairs. The introduced interdependencies between the different country pairs make

[24] The model becomes numerically unstable for implied local curvatures above 25. We therefore cannot report model solutions for higher levels.

Table III.7: UIP slope coefficient — simultaneous calibration

Ctry1	Ctry2	$\mathbb{E}[h_1]$	$\sigma[h_1]$	$\mathbb{E}[h_2]$	$\sigma[h_2]$	Model β	Emp. β	s.e.
AU	DE	0.86	0.0034	0.95	0.0041	-0.05	0.25	[0.21]
AU	JP	0.86	0.0034	0.88	0.0051	0.10	0.10	[0.27]
AU	UK	0.86	0.0034	0.95	0.0044	-0.07	-0.19	[0.23]
AU	US	0.86	0.0034	0.97	0.0038	0.17	-0.04	[0.17]
DE	JP	0.95	0.0041	0.88	0.0051	-0.30	0.12	[0.29]
DE	UK	0.95	0.0041	0.95	0.0044	-0.04	0.27	[0.18]
DE	US	0.95	0.0041	0.97	0.0038	-0.11	-0.03	[0.21]
JP	UK	0.88	0.0051	0.95	0.0044	-0.32	-1.05	[0.38]
JP	US	0.88	0.0051	0.97	0.0038	-0.27	-0.63	[0.25]
UK	US	0.95	0.0044	0.97	0.0038	-0.15	-0.04	[0.19]

This table reports the empirical vs. model implied slope coefficient for the calibration case: one habit parametrization per country. The first two columns refer to the country pair. The next two columns to the first two habit moments of country 1. The next two columns to the habit parameterization of country 2. Finally, the last three columns compare the model implied value to the empirical observation.

the calibration computationally much more complex.

The deviations of the slope coefficient are obviously larger than in the separate calibration. Nevertheless, every country pair remains within two standard deviations. The results from this table suggest that Americans have the highest habit level (0.97). They are closely followed by the Europeans, Germans (0.95) and British (0.95) also display relatively high habit levels. The two countries from the far east, Japan (0.88) and Australia (0.86), show much lower habit levels.

Joint calibration — the same habit parameterization for everybody

In the final calibration exercise, we want to analyze the model's performance in the most stringent cross-country setup. Table III.8 displays the results, when we restrict the habit parameterization to be the same across all countries. The closest fit is attained for an average habit of $\mathbb{E}[h] = 0.96$ and a habit volatility of $\sigma[h] = 0.0024$. The lack of flexibility obviously results in much larger deviations of the model implied values to the actual data. While the joint calibration fails to account for JP - UK and DE - UK, the fit is acceptable for eight out of ten country pairs remaining within two standard deviations of the data.

III.6 Conclusion

This paper studies the co-movement between interest rates and exchange rates within a Lucas (1982) style model with endogenous consumption decisions. The most crucial additional assumptions are habit formation, incomplete markets and country-specific goods.

Table III.8: UIP slope coefficient — joint calibration

Ctry1	Ctry2	Model β	Emp. β	s.e.
AU	DE	-0.05	0.25	[0.21]
AU	JP	0.30	0.10	[0.27]
AU	UK	-0.04	-0.19	[0.23]
AU	US	0.07	-0.04	[0.17]
DE	JP	-0.20	0.12	[0.29]
DE	UK	-0.17	0.27	[0.18]
DE	US	0.37	-0.03	[0.21]
JP	UK	-0.16	-1.05	[0.38]
JP	US	-0.18	-0.63	[0.25]
UK	US	0.35	-0.04	[0.19]

This table shows the empirical vs. model implied slope coefficient for the calibration case: the same habit parameterization for every country ($\mathbb{E}[h] = 0.96$, $\sigma[h] = 0.0024$).

Theoretically, risk premia drive time-varying international hedging decisions, which lead to a possible explanation for the forward premium anomaly.

Empirically, the model convinces twofold. Firstly, it matches the first two moments for FX returns, inflation, income growth; and most notably the correlation between FX returns and income growth. Secondly, it reproduces the slope coefficient in the regression of FX returns on interest rate differentials for ten different country pairs.

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III.A Robustness in technical parameters

Table III.9: Technical parameters

Parameter (Quarterly)		Value
Habit boundaries scale	ζ	1
Habit grid size	n_h	3
Wealth boundaries	$\bar{w}^H = \bar{w}^F$	-0.1
Wealth grid size	n_w	11

In addition to our economic calibration, there are also a few technical parameters, which we need to choose for the numerical procedure. These parameters are listed in Table III.9. The parameter choice reflects a trade-off between computational effort and accuracy. The idea of this section is to show that our main model result, the UIP slope coefficient, does not dramatically change in any of these parameters.

Habit discretization

We discretize habit in the following fashion. Adding (subtracting) habit volatility times the habit boundary scale factor ζ from average habit yields the upper (lower) bound for the habit grid. Given the boundaries, we construct a linearly spaced grid with n_h points. Since it is convenient to have average habit as a gridpoint, we restrict the number of gridpoints to an uneven number,

Figures III.5a and III.5b display the change in the slope coefficient when varying these parameters. The number of grid points has almost no impact while the scaling factor has a slight impact on the slope coefficient. Different discretization change the actual volatility of habit resulting in different slope coefficients. Since habit volatility is calibrated to fit the UIP slope, this lack of robustness is not a major issue. It only limits the comparability of the absolute level of habit volatility across different numbers of habit nodes (n_h).

Wealth discretization

Wealth is also discretized on an equally spaced grid. The boundaries are set to $\bar{w}$. n_w determines the number of grid point. Figures III.5c and III.5d clearly show that both parameters have no major impact on our model result.

III.B First order conditions

III.B.1 Normalization

To solve our model we first rewrite the nominal problem (eq. III.4) into the corresponding real problem. For this purpose we set price level of nontradables to 1 and the price level of tradables as p_1 respectively p_2 for each country.

Let us denote $R_{1,t} = \frac{1}{1+\pi_t^H}$ and $R_{2,t} = \frac{1}{1+\pi_t^F}$ as the real returns of each bond. Furthermore we redefine the shock vector and state space in real terms as follows: $z_t = (R_{1,t}, R_{2,t}, y_t^H, y_t^F)$ and $\Psi_t = (w_t, z_t, h_t)$. Then the dynamic programming problem transforms into

$$V_t(\Psi_t) = \underset{c_t, b_{1,t}, b_{2,t}}{max} u(c_t, h_t) + \delta\, \mathbb{E}_t[V_{t+1}(\Psi_{t+1})],$$

subject to

$$\begin{aligned}
c_{TG,t} &\leq w_t + y_{TG,t} - q_{1,t}b_{1,t} - q_{2,t}b_{2,t}, \\
c_{NG,t} &\leq y_{NG,t}, \\
w_{t+1} &= R_{1,t+1}b_{1,t} + R_{2,t+1}b_{2,t}, \\
b_{1,t} &\geq \frac{\bar{b_1}}{E[R_{1,t+1}]}, \\
b_{2,t} &\geq \frac{\bar{b_2}}{E[R_{2,t+1}]}, \\
\bar{w} &\leq b_{1,t}q_{1,t} + b_{2,t}q_{2,t}, \\
c_{TG,t} &\geq h_{TG,t}.
\end{aligned}$$

The last inequality is unnecessary in theory. The utility function is simply not defined for values smaller than 0. However, it is necessary to enforce the condition for computational reasons, as a solver might try to evaluate the function for $c_{TG,t} < h_{TG,t}$. Depending on the choice of the risk aversion, this could either result in complex numbers or even lead to a potential solution of the equation system, with no economic meaning.

To facilitate computation, we additionally normalize each agent's problem with factors κ^A and κ^B respectively. Given homothetic preferences the individual's policies are simply scaled by the normalization factor. Thus, the equilibrium remains unchanged under the appropriate adjustment of market clearing conditions (see next section).
Define $\tilde{z}_t = (R_{1,t+1}, R_{2,t+1}, \tilde{y}_t^H, \tilde{y}_t^F)$ and $\tilde{\Psi}_t = (\tilde{w}_t, \tilde{z}_t, \tilde{h}_t)$ where $\kappa\tilde{y}_t = y_t$, then

$$V_t(\tilde{\Psi}_t) = \underset{\tilde{c}_t, \tilde{b}_{1,t}, \tilde{b}_{2,t}}{max} u(\tilde{c}_t, \tilde{h}_t) + \delta\, \mathbb{E}_t[V_{t+1}(\tilde{\Psi}_{t+1})],$$

subject to

$$
\begin{aligned}
\tilde{c}_{TG,t} &\leq \tilde{w}_t + \tilde{y}_{TG,t} - q_{1,t}\tilde{b}_{1,t} - q_{2,t}\tilde{b}_{2,t}, \\
\tilde{c}_{NG,t} &\leq \tilde{y}_{NG,t}, \\
\tilde{w}_{t+1} &= R_{1,t+1}\tilde{b}_{1,t} + R_{2,t+1}\tilde{b}_{2,t}, \\
\tilde{b}_{1,t} &\geq \frac{\tilde{\bar{b}}}{E[R_{1,t+1}]}, \\
\tilde{b}_{1,t} &\geq \frac{\tilde{\bar{b}}_1}{E[R_{1,t+1}]}, \\
\tilde{b}_{2,t} &\geq \frac{\tilde{\bar{b}}_2}{E[R_{2,t+1}]}, \\
\tilde{\bar{w}} &\leq \tilde{b}_{1,t}q_{1,t} + \tilde{b}_{2,t}q_{2,t}, \\
\tilde{c}_{TG,t} &\geq \tilde{h}_{TG,t}
\end{aligned}
$$

III.B.2 Kuhn-Tucker conditions

Concavity of the utility function allows us to impose equality for the first two conditions. Inserting conditions two and three and denoting for simplicity $u(c_t, h_t) = u(c_{TG,t})$ we can write the Lagrangian as

$$
\begin{aligned}
\mathcal{L} = u(\tilde{c}_{TG,t}) + \delta\, \mathbb{E}_t[V_{t+1}(\tilde{\Psi}_{t+1})] \; &+ \; \mu \; (\tilde{y}_{TG,t} + \tilde{w}_t - q_{1,t}\tilde{b}_{1,t} - q_{2,t}\tilde{b}_{2,t} - \tilde{c}_{TG,t}) \\
&+ \; \lambda_1 \; (\tilde{b}_{1,t}E_t[R_{1,t+1}] - \tilde{\bar{b}}_1) \\
&+ \; \lambda_2 \; (\tilde{b}_{2,t}E_t[R_{2,t+1}] - \tilde{\bar{b}}_2) \\
&+ \; \lambda_3 \; (\tilde{b}_{1,t}q_{1,t} + \tilde{b}_{2,t}q_{2,t} - \tilde{\bar{w}}) \\
&+ \; \lambda_{nn} \; (\tilde{c}_{TG,t} - \tilde{h}_{TG,t}).
\end{aligned}
$$

Deriving the Lagrangian with respect to each choice variable, adding the conditions and restrictions on the Lagrange multipliers provides us with the following system of first order conditions:

$$\frac{\partial \mathcal{L}}{\partial \tilde{c}_{TG,t}} = u'(\tilde{c}_{TG,t}) - \mu + \lambda_{nn} \stackrel{!}{=} 0,$$

$$\begin{aligned}
\frac{\partial \mathcal{L}}{\tilde{b}_{1,t}} &= \delta \mathbb{E}_t \left[\frac{\partial V_{t+1}}{\partial \tilde{b}_{1,t}} \right] - \mu q_{1,t} + \lambda_1 E[R_{1,t+1}] + \lambda_3 q_{1,t} \\
&= \delta \sum_{z_{t+1} \in \Gamma(z_t)} \left[\pi(z_{t+1}|z_t) \frac{\partial u}{\partial \tilde{c}_{TG,t+1}} \frac{\partial \tilde{c}_{TG,t+1}}{\partial \tilde{w}_{t+1}} \frac{\partial \tilde{w}_{t+1}}{\partial \tilde{b}_{1,t}} \right] - \mu q_{1,t} + \lambda_1 E[R_{1,t+1}] + \lambda_3 q_{1,t} \\
&= \delta \sum_{z_{t+1} \in \Gamma(z_t)} \left[\pi(z_{t+1}|z_t) \frac{\partial u}{\partial \tilde{c}_{TG,t+1}} \frac{\partial \tilde{c}_{TG,t+1}}{\partial \tilde{w}_{t+1}} R_{1,t+1} \right] - \mu q_{1,t} + \lambda_1 E[R_{1,t+1}] + \lambda_3 q_{1,t} \stackrel{!}{=} 0, \\
\frac{\partial \mathcal{L}}{\tilde{b}_{2,t}} &= \delta \mathbb{E}_t \left[\frac{\partial V_{t+1}}{\partial \tilde{b}_{2,t}} \right] - \mu q_{2,t} + \lambda_2 + \lambda_3 q_{2,t} \\
&= \delta \sum_{z_{t+1} \in \Gamma(z_t)} \left[\pi(z_{t+1}|z_t) \frac{\partial u}{\partial \tilde{c}_{TG,t+1}} \frac{\partial \tilde{c}_{TG,t+1}}{\partial \tilde{w}_{t+1}} \frac{\partial \tilde{w}_{t+1}}{\partial \tilde{b}_{2,t}} \right] - \mu q_{2,t} + \lambda_2 E[R_{2,t+1}] + \lambda_3 q_{2,t} \\
&= \delta \sum_{z_{t+1} \in \Gamma(z_t)} \left[\pi(z_{t+1}|z_t) \frac{\partial u}{\partial \tilde{c}_{TG,t+1}} \frac{\partial \tilde{c}_{TG,t+1}}{\partial \tilde{w}_{t+1}} R_{2,t+1} \right] - \mu q_{2,t} + \lambda_2 E[R_{2,t+1}] + \lambda_3 q_{2,t} \stackrel{!}{=} 0,
\end{aligned}$$

$$\begin{aligned}
\lambda_1(\tilde{b}_{1,t} E[R_{1,t+1}] - \bar{\tilde{b}}_1) &= 0, \\
\lambda_2(\tilde{b}_{2,t} E_t[R_{2,t+1}] - \bar{\tilde{b}}_2) &= 0, \\
\lambda_3(\tilde{b}_{1,t} q_{1,t} + \tilde{b}_{2,t} q_{2,t} - \bar{w}) &= 0 \\
\lambda_{nn}(\tilde{c}_{TG,t} - (1-\eta) h_t) &= 0,
\end{aligned}$$

$$\lambda_1 \geq 0, \quad \lambda_2 \geq 0, \quad \lambda_3 \geq 0, \quad \lambda_{nn} \geq 0$$

where $\Gamma(z_t)$ denotes all states possibly following z_t and $\pi(z_{t+1}|z_t)$ are the transition probabilities.

The same set of equations exists for the second agent and is completed by the market clearing conditions

$$\begin{aligned}
\kappa^A \tilde{b}^A_{1,t} + \kappa^B \tilde{b}^B_{1,t} &= 0, \\
\kappa^A \tilde{b}^A_{2,t} + \kappa^B \tilde{b}^B_{2,t} &= 0, \\
\kappa^A \tilde{c}^A_{TG,t} + \kappa^B \tilde{c}^B_{TG,t} &= \kappa^A \tilde{y}^A_{TG,t} + \kappa^B \tilde{y}^B_{TG,t}.
\end{aligned}$$

The market clearing conditions apply to the unnormalized economy. Thus, terms are unnormalized with the agent specific normalization coefficient.

III.B.3 Alternative conditions

It is computationally inconvenient to work with the inequality constraints for the Lagrange multiplier. Therefore we use the following reformulation as described in Zangwill and Garcia (1981).

The key is to replace the Lagrange multipliers by slacks, which are decomposed into a positive and negative part

$$\alpha^{+} = [\ max(0, \alpha)\ ]^{k},$$
$$\alpha^{-} = [\ max(0, -\alpha)\ ]^{k}.$$

One would expect a k of 2 or 3 to work best to avoid any kinks in the nonlinear system of equations. However, surprisingly, we find that $k = 1$ outperforms any other choice.

This allows us to rewrite the first order conditions into the following equivalent system

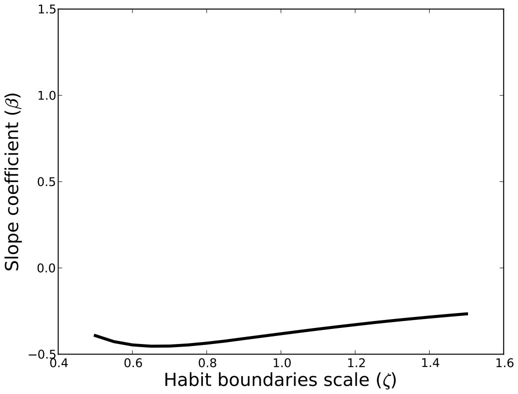
$$\begin{aligned}
\frac{\partial \mathcal{L}}{\partial \tilde{c}_{TG,t}} &= u'(\tilde{c}_{TG,t}) - \mu + \alpha_{nn}^{+} + && \stackrel{!}{=} 0,\\
\frac{\partial \mathcal{L}}{\tilde{b}_{1,t}} &= \delta \sum_{z_{t+1} \in \Gamma(z_t)} \left[\pi(z_{t+1}|z_t) \frac{\partial u}{\partial \tilde{c}_{TG,t+1}} R_{1,t+1} \right] - \mu q_{1,t} + \alpha_1^{+} E_t[R_{1,t+1}] + \alpha_3^{+} q_{1,t} && \stackrel{!}{=} 0,\\
\frac{\partial \mathcal{L}}{\tilde{b}_{2,t}} &= \delta \sum_{z_{t+1} \in \Gamma(z_t)} \left[\pi(z_{t+1}|z_t) \frac{\partial u}{\partial \tilde{c}_{TG,t+1}} R_{2,t+1} \right] - \mu q_{2,t} + \alpha_2^{+} E_t[R_{2,t+1}] + \alpha_3^{+} q_{2,t} && \stackrel{!}{=} 0,
\end{aligned}$$

$$\begin{aligned}
\alpha_1^{-} &- (\tilde{b}_{1,t} E[R_{1,t+1}] - \tilde{\tilde{b}}_1) && = 0,\\
\alpha_2^{-} &- (\tilde{b}_{2,t} E_t[R_{2,t+1}] - \tilde{\tilde{b}}_2) && = 0,\\
\alpha_3^{-} &- (\tilde{b}_{1,t} q_{1,t} + \tilde{b}_{2,t} q_{2,t} - \bar{w}) && = 0,\\
\alpha_{nn}^{-} &- (\tilde{c}_{TG,t} - (1-\eta) h_t) && = 0,
\end{aligned}$$

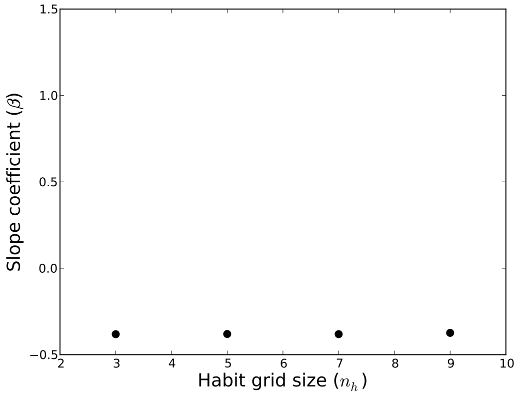
α can be interpreted as the shadow price of the borrowing constraint. If the constraint does not bind then α is negative and α^{-} positive which equalizes the $\geq$ constraint. Thus, essentially the borrowing constraint does not have a shadow price. If α is positive α^{+} is positive showing up in the FOCs while the borrowing constraint exactly binds. The higher α the more costly is the constraint.

Figure III.5: UIP slope coefficient over various technical parameters

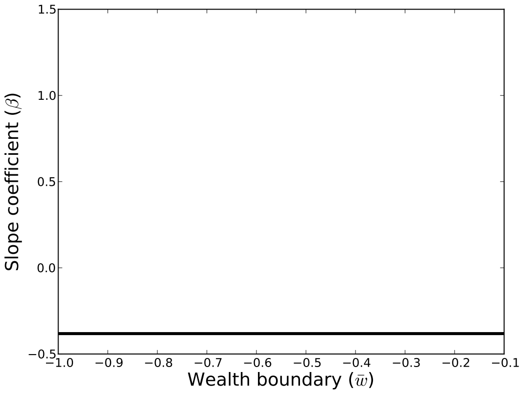
(a) Habit boundaries



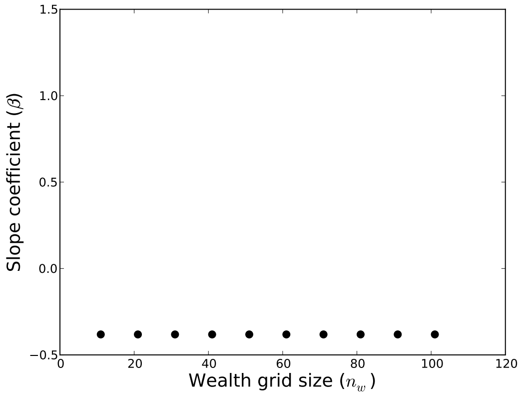
(b) Habit grid size



(c) Wealth boundary



(d) Wealth grid size



These figures report robustness checks in four technical parameters. We show how changes in these parameters affect the slope coefficient in the base calibration.

Part IV

Calibrating/Estimating Economic Models Using Parallel Computing

Calibrating/Estimating Economic Models using Parallel Computing

Benjamin Jonen*

In this writeup, I survey various techniques used to pin down parameters in macroeconomic models. I look at parameter choice in dynamic stochastic general equilibrium models and focus on the way parallel computing can play an important role both in calibration and estimation. Finally, I conclude with a specific application of parallel computing to calibrate the model in Jonen and Scheuring (2012).

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IV.1 Introduction

In this paper I provide an overview of calibration and estimation techniques used to parameterize macroeconomic models, particularly dynamic stochastic equilibrium (DSGE) models.

Macroeconomic models become increasingly complex. Analytical solutions are rarely available so that researchers have to rely on numerical solutions to analyze their models. Numerical dynamic programming is a common avenue to generate model solutions. The algorithm recursively solves two-period problems until the policy (or value) functions converge. Use of computational methods requires the discretization of the state space, usually approximating the continuous state process with a discrete Markov chain, and the approximation of the target functions.

Once a model solution is available, the next step is to assess the validity of the model. The question of what makes a good model and how to assess its quality is controversially discussed in the literature. I will provide a summary of the debate in the next section.

One common approach in the literature is to *calibrate* the model. First, a set of parameters is estimated from data or taken from existing studies. For example, preference parameters can sometimes be taken from research in experimental economics. Second, the parameters which cannot be easily observed have to be chosen in a meaningful way. This step gives the researcher considerable degree of freedom which has to be used with caution. Usually the parameters are chosen to match some model-implied moments.

Calibration is often criticized to lack a solid statistical foundation. Advocates of *estimation* are looking for methods to apply econometric techniques to estimate model parameters and test the model's validity. Various estimation techniques exist and are specifically tailored towards the estimation of DSGE models.

By nature of numerical solutions, it is impossible to evaluate the model at all parameter combinations. However, depending on the type of problem, the model has to be evaluated at a large number of parameter combinations, either to pin down the free parameters, maximize some likelihood or even to perform sensitivity analysis. Given today's state of computers, parallelization is the natural way to speed up the evaluation process. Parallelization can be done at different levels of the algorithm. I will argue that for many macroeconomic models the parallelization should be done at the parameter level.

The paper concludes with a concrete example of how a global optimization routine that dispatches jobs on a grid computer is used to calibrate the model proposed in Jonen and Scheuring (2012).

IV.2 Calibration

Macroeconomic modeling has undergone large developments over the last decades. DeJong and Dave (2011) summarize that system-of-equation models and in particular models of behavioral equations "represented state-of-the-art practice in econometrics into the 1970s." These models, however, had one major shortcoming: In his influential paper Robert E. Lucas (1976) points out that model parameters describe the relation between macroeconomic variables under certain policy regimes and are thus potentially sensitive to the studied policy shifts. In his concluding remarks Robert E. Lucas (1976) notes that

> "given that the structure of an econometric model consists of optimal decision rules of economic agents, and that optimal decision rules vary systematically with changes in the structure of series relevant to the decision maker, it follows that any change in policy will systematically alter the structure of econometric models. "

As a reaction to this critique, macroeconomists have focused their attention on DSGE models. In contrast to the above system-of-equation models, DSGE models have a microeconomic foundation: The basic structure involves the optimization of one or many decision makers interacting on markets that are assumed to clear in equilibrium. The majority of model parameters describe the preferences of individuals and the hope is that such parameters remain unchanged in policy regime shifts. This appealing class of models comes at a cost however - models become increasingly hard to solve and assess empirically. Estimation of DSGE models generally involve the reformulation of the model's nonlinear system of equation into a state-space representation. Then the derivation of the model's likelihood function, finally the application of maximum likelihood to arrive at parameter estimates. When state-transition and measurement equations are linear (usually the result of a log-linearization of a DSGE model) and innovations are normally distributed, the likelihood can be derived analytically using the Kalman filter. Otherwise numerical methods have to be used to evaluate the integrals that characterize the likelihood function (see DeJong and Dave (2011) chapter 8).

Kydland and Prescott (1982) represents a turning point in the empirical assessment of macroeconomic models. The authors renounce a formal econometric test or estimation of the model and rather pick some parameters according to accounting data or micro studies[1] while considering the remaining parameters as "free." They evaluate the model on a grid for the free parameter values and then choose the parameter values "that yielded what [they] considered to be the best fit." The resulting best fit is subjectively judged as "very good, particularly in light of the model's simplicity," while no formal statistical test of the model is conducted. Lucas (1987) argues that it is obvious the model is not 'true' and that the best modelers can strive for "is a workable approximation that is useful in answering a limited set of questions."

The debate around calibration has been controversial. Many proponents of estimation regard calibration, at best, as a first step in model assessment. Hansen and Sargent (1988) write "the calculations described here are intended as a prolegomenon to pursuing estimation, and as a way of indicating whether the model might match the data," similarly Manuelli and Sargent (1988) write "what reasons are there to expect the informal metric implicitly used by Kydland-Prescott to yield better (in what sense?) estimators than estimators derived from a formal metric [...]." According to Hansen and Heckman (1996) the

[1] A process Hoover (1995) refers to as "casual empiricism."

calibration methodology consists of two steps - calibration and verification which corresponds to the standard econometric practice of estimation and testing. They conclude that the calibration methodology is merely the use of a non-standard, "implicit" loss function. "In looking at economic aggregates, [business cycle practicioners'] implicit loss function appear to focus on the model predictions for long-run means, to the exclusion of other features of the data, when selecting parameter estimates." Hansen and Heckman further criticize the casual use of microeconomic research as a basis for macroeconomic parameter estimates: They argue that macroeconomists often use parameter estimates without taking model-specification error as well as estimation error into account. If the estimates have been obtained under the assumption of a different economic environment (model), they might need some form of adjustment to serve as macroeconomic inputs. Standard errors for parameter inputs are rarely reported and many papers provide insufficient sensitivity analyses.

In the mid 90s Kydland and Prescott (1996) conclude that computational experiments (model calibration) "have become invaluable tools for quantitative aggregate theory", and give a five step approach to execute a computational experiment. Step one and two are to pose a question and to find a "well-tested theory" that suits this question. Step three is the actual construction of the model to be computed. The fourth, is the controversial step of model calibration. In this step "data are used to calibrate the model economy so that it mimics the world as closely as possible along a limited, but clearly specified, number of dimensions." Once calibrated the model is then able to provide insights about outcomes under different policy regimes. The final step is to run the computational experiment and compare the simulated to the real world data. Hoover (1995), on the other hand, remains doubtful. He writes that "above all, it is not clear on what standards competing, but contradictory models are to be compared and adjudicated [within the calibration framework]."

While calibration remains an important tool in macroeconomics today, especially to get a first impression of the model's quality, several estimation techniques have matured and endow researchers with a number of techniques to estimate DSGE models. I will summarize several of these techniques in the next section. The discussion is primarily based on DeJong and Dave (2011).

IV.3 Estimation

In this section I briefly review the main estimation techniques for DSGE models, broadly categorizing them into likelihood based and moment based estimation.

IV.3.1 Likelihood based methods

The first step in likelihood based estimation is to rewrite the nonlinear equations of a DSGE model into a state-space representation. I lay out the reformulation following DeJong and Dave (2011) chapter 8.3. Let y_t be a vector of observable variables, where $\Psi_t \equiv \{y_j\}_{j=1}^{t}$.

Also let s_t be a vector of unobserved state variables with $s_t \equiv \{s_j\}_{j=1}^t$. The state-space representation can be characterized by the state-transition density

$$f(s_t|s_{t-1}, \Psi_{t-1}),$$

the observation density

$$f(y_t|s_t, \Psi_{t-1}),$$

and the initial distribution of the state

$$f(s_0) \equiv f(s_0|\Psi_0).$$

After the DSGE model has been cast into this representation, the time-t likelihood function can be computed from

$$f(y_t|\Psi_{t-1}) = \int f(y_t|s_t, \Psi_{t-1}) f(s_t|\Psi_{t-1}) ds_t, \tag{IV.1}$$

where the so called predictive density can be obtained from the state-transition density by marginalizing over s_{t-1}

$$f(s_t|\Psi_{t-1}) = \int f(s_t|s_{t-1}, \Psi_{t-1}) f(s_{t-1}|\Psi_{t-1}) d_{s_{t-1}}. \tag{IV.2}$$

and the so called filtering density is given by

$$f(s_t|\Psi_t) = \frac{f(y_t|s_t, \Psi_{t-1}) f(s_t|\Psi_{t-1})}{f(y_t|\Psi_{t-1})}. \tag{IV.3}$$

Given the dependencies in equations (IV.1), (IV.2) and (IV.3), the computation of the time-t likelihood function has to be performed sequentially, starting from the filtering density for $t = 1$

$$f(s_1|\Psi_0) = \int f(s_1|s_0, \Psi_0) f(s_0) d_{s_0}.$$

In general the integral in eq. (IV.1) is hard to solve analytically. In chapter 9 of their book DeJong and Dave (2011) discuss Monte Carlo integration techniques to assess the value of this integral numerically. The overall log-likelihood function associated with a parameter vector μ is given by

$$logL(\Psi_T|\mu) = \sum_{t=1}^{T} f(y_t|\Psi_{t-1}).$$

Amemiya (1985) shows that under regularity conditions the maximum likelihood estimator is consistent and asymptotically normal. This is the classical way to estimate and conduct inference for DSGE models.

An alternative to maximum likelihood estimation are Bayesian methods. Bayesian methods appear well-suited for DSGE estimation because researchers usually have some prior on the relevant parameter range of their structural, as opposed to reduced form, models. Ruge-Murcia (2007) writes "economic theory, previous macroeconomic studies and long-run averages of aggregate data can be informative about the parameter values in structural macroeconomic models." In calibration, this prior is used to pin down the majority of parameters directly, while under Bayesian estimation the researcher states a prior distribution of the parameters. While maximum likelihood relies on the maximization of a highly dimensional complicated function, Fernández-Villaverde (2010) points out that Bayesian analysis relies on integrating that function, which is arguably easier.

IV.3.2 Moment based estimation

Similar to the calibration methodology, moment based estimation focuses on key aspects (moments) of the data instead of targeting an overall fit. Hoover (1995) notes that "calibrators of real-business-cycle models typically concentrate on matching selected second moments of variables rather than, say, matching the actual historical evolution of the modeled variables" because the focus is on the model's ability to fit one time series but to characterize the "distribution from which that realization was drawn." In contrast to the calibration methodology, however, moment based estimation is a formal econometric technique that allows for estimation and formal statistical testing.

In cases when the target moments can be evaluated analytically the generalized method of moments (GMM) described in Hansen and Singleton (1982) can be applied. This is generally not the case for DSGE models. I will thus focus my attention on the so called method of simulated moments (SMM) which applies the ideas of GMM to simulated data.

SMM was first developed in the context of discrete choice (see McFadden (1989) and Pakes and Pollard (1989)) and later extended to time series models (see Lee and Ingram (1991) and Duffie and Singleton (1993)). The key problem to apply the method of moments to the simulated time series data of DSGE models is that the data itself depends on the parameter choice. That is, while the standard Hansen GMM objective depends on the parameters directly, model simulated data depends on the parameters also through the time series itself. Duffie and Singleton derive conditions under which the SMM estimator can be used. Additionally they prove consistency and characterize estimator's distribution. In the following, I define the SMM estimator and its distribution closely following Duffie and Singleton (1993) and Ruge-Murcia (2007).

We are confronted with real world data y_t of T data points. Additionally, for any given parameter vector μ, simulate the model economy and obtain the time series y_t^μ with τT data points, where the introduction of τ allows us to simulate a longer time series than the actually observed series. Given this data we choose an observation function f that describes the moments of the data we are interested in. The function $f_t^* = f(y_t, y_{t-1}, ..., y_{t-l+1})$ considers the past l ($l < T$) observations and generates actual data moments. The counterpart $f_t^\mu = f(y_t^\mu, y_{t-1}^\mu, ..., y_{t-l+1}^\mu)$ describes the moments for the simulated data at parameter μ.

For given μ, the difference in sample moments is defined in Duffie and Singleton (1993) as

$$G_T(\mu) = \frac{1}{T}\sum_{t=1}^{T} f_t^* - \frac{1}{\tau T}\sum_{s=1}^{\tau T} f_s^\mu$$

The SMM estimator is the value of μ that minimizes the quadratic form

$$\mu_{SMM} = \underset{\mu}{\operatorname{argmin}}\, G_T(\mu)' W G_T(\mu), \tag{IV.4}$$

where W is the optimal weighting matrix

$$W = \lim_{T\to\infty} Var((1/\sqrt{T})\sum_{t=1}^{T} f_t^\mu).$$

Under the regularity conditions in Duffie and Singleton (1993) (section 5), the authors show that the SMM estimator is asymptotically normal

$$\sqrt{T}(\mu_{SMM} - \mu) \to N(0, (1 + 1/\tau)(D'W^{-1}D),$$

where $D = \mathbb{E}(\partial f_t^\mu / \partial \mu)$. For given μ, D can be approximated as

$$D(\mu) = \frac{\partial[\frac{1}{\tau T}\sum_{t=1}^{\tau T} f_t^\mu]}{\partial \mu}.$$

IV.4 Empirical example

In this section, I describe the calibration strategy in Jonen and Scheuring (2012). The strategy is close to the method of simulated moments, but with a target function that punishes deviations more strongly. As we move away from SMM, we give up the ability to formally test our model and its parameters. Instead we focus on finding parameters for which the deviation between model predictions and data is below a certain cutoff across different countries simultaneously. I now proceed to give a detailed summary of the calibration strategy employed in the paper, relating it to the discussion in the previous sections.

We develop a two-country model that allows us to replicate the empirical observation that interest rate differentials have no or reverse predictability of exchange rate returns. The model is calibrated using data estimates, some self-computed, some taken from the literature, leaving open only the parametrization of the habit process

$$h_{t+1} = \rho h_t + \eta c_t. \tag{IV.5}$$

To make the calibration procedure more intuitive, we choose the first two moments of the process, $\mathbb{E}[h]$ and $\mathbb{V}[h]$. The parameters ρ and η can be backed out using

$$\rho = \frac{\frac{\mathbb{E}[h]^2}{\mathbb{E}[c]^2}\mathbb{V}[c] - \mathbb{V}[h]}{\frac{\mathbb{E}[h]^2}{\mathbb{E}[c]^2}\mathbb{V}[c] + \mathbb{V}[h]}, \tag{IV.6}$$

$$\eta = (1-\rho)\frac{\mathbb{E}[h]}{\mathbb{E}[c]},$$

where $\mathbb{E}$ refers to the unconditional expectation and $\mathbb{V}$ to the unconditional variance.

Our goal is to find a parametrization for the habit process that resolves the forward premium puzzle for five different countries. Given our two-country model economy that means we analyze ten different country pairs. To make a convincing cross-country calibration, each country maintains the same habit parametrization independent of the country it interacts with. Our general calibration strategy is to find parameters to minimize the distance between model prediction and data. Two parameters for each country determine preferences in the model. Thus we face a ten-dimensional optimization problem. One possible objective would be to minimize the squared deviation between model predicted and empirical slope coefficient. This objective is similar to the SMM objective in eq. (IV.4), where $W = I$ and the moments $\frac{1}{S}\sum_{s=1}^{S} f_t$ are replaced with $\frac{Cov(\Delta s_{t+1}, i_t - i_t^*)}{V(\Delta s_{t+1}}$, the slope coefficient of a forward premium regression. Instead of simply minimizing the distance in slopes, we look at the distance in slopes scaled by the standard error of the empirical regression coefficient

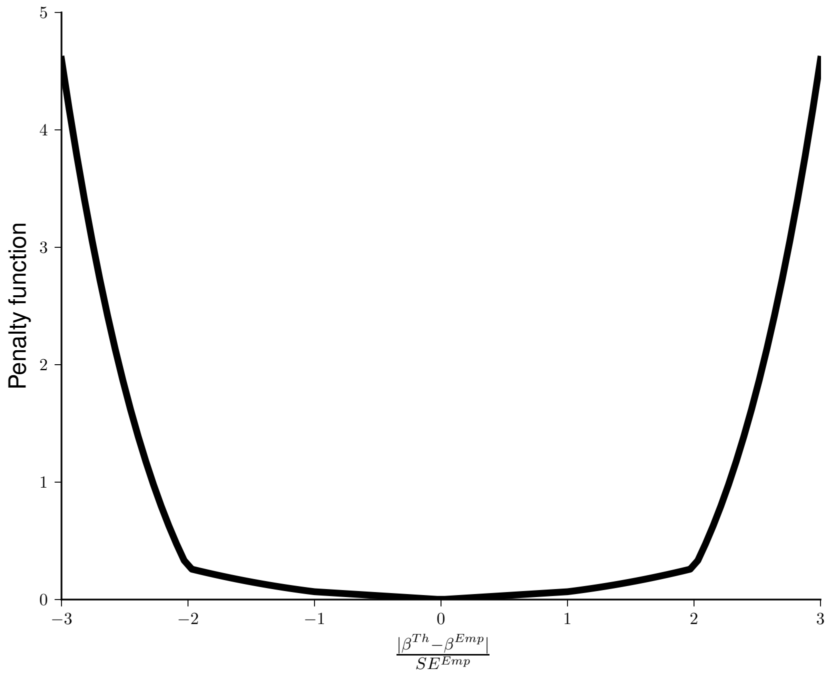
$$\frac{|\beta^{Th} - \beta^{Emp}|}{SE^{Emp}},$$

where β^{Th} and β^{Emp} represent the slope coefficients obtained from forward premium regressions of the simulated and actual data respectively and SE^{Emp} represents the standard error of the slope coefficient of the regression on actual data. In the optimization procedure we additionally weight each country-pair's deviation to arrive at a parametrization that fits all country pairs relatively well. The penalty function is plotted in Figure IV.1. Deviations up to one standard error are weighted linearly, while deviations between one and two standard errors are weighted quadratically. Deviations beyond two standard errors are weighted with power four.

We use a global optimization routine to solve this ten-dimensional non-convex problem. In particular we use a differential evolution algorithm described in Storn and Price (1997). Figure IV.2 summarizes the general idea in the following simplified way: Draw an initial population and evaluate the population with the objective function. In the next step the algorithm generates a new population according to some "evolution" rule. After the new population is evaluated, the superior population members survive. If the surviving population fulfills the convergence criterion, the optimization is terminated with success otherwise the process starts over again.

The complexity of the problem requires a large number of model solutions. But since model evaluations at different parameter combinations are independent of each other, we have the possibility to execute them in parallel. In contrast to gradient based solvers for example, the critical feature of the differential evolution algorithm is that evaluations within one iteration are independent of each other. This is depicted in the second panel of Figure IV.2. Instead of evaluating one population member after the other we can evaluate all of them in parallel on a grid computer. Of course the next step in which populations

Figure IV.1: Penalty function for each country-pair deviation



This figure displays the penalty function employed in the optimization procedure. We impose small punishment on deviations smaller one, larger punishment up to a deviation of two and quickly growing punishment for larger deviations.

compete is delayed until all evaluations have completed. In our case this is hardly a constraint as model evaluation times are quite homogeneous.

To make use of these observations, we integrated the differential evolution implementation from Chakraborty (2008) into the grid computing software gc3pie[2], a software, quoting from its web site, that "aids in submitting and controlling batch jobs to clusters and grid resources seamlessly." With this implementation of the global optimization routine, we are able to perform the evaluation steps in parallel submitting jobs to the Swiss-wide cluster network available through the Grid Computing Competence Center at the University of Zurich.

Figure IV.3 displays the populations development over algorithm iterations. Panel IV.3a shows the initial random sample of 100 parameter combinations. Parameters are drawn within the following constraints. First we impose upper and lower bounds on the parameters. Average habit is constrained to lie in the range 0.8 to 1.0, while habit volatility is constrained to lie between 0.000 and 0.010. Additionally we impose

$$\sigma_H \leq \frac{\mu_H}{\mu_Y}\sigma_Y,$$

making sure that tomorrow's habit is a positive function of today's consumption. These constraints are country-dependent, and are depicted in Figure IV.3 with the respective country's color. Note that each parameter combination lies below the respective country's habit volatility constraint and within the box constraints. The figure shows how the population evolves over time. After 40 iterations the parameter combinations already start to cluster. The process continues for the next 120 iterations. After 240 iterations the population of parameter combinations yields a clear indication of the model implied habit parameters for all countries except Australia. UK has the lowest habit volatility. Average habit is higher in the US than in Germany and higher in Germany than in Japan. Because of this distinct clustering and the optimizer's inability to improve the best target value, the population is likely to remain unchanged and we terminate the optimization.

The general structure of this application is very similar to the estimation techniques described in the previous sections. While we do not perform formal statistical testing we move away from picking (potentially by hand) some parameter combination that fits the data well and instead explicitly employ a target (loss) function.

Other techniques, for example the method of simulated moments, may easily be cast in the computational framework that we constructed. Similar to this application the SMM objective function is usually non-convex and requires the use of a global optimization routine. With the help of current grid computing, these problems can be solved in reasonable time allowing the researcher to estimate structural model parameters and importantly report measures of certainty about these values.

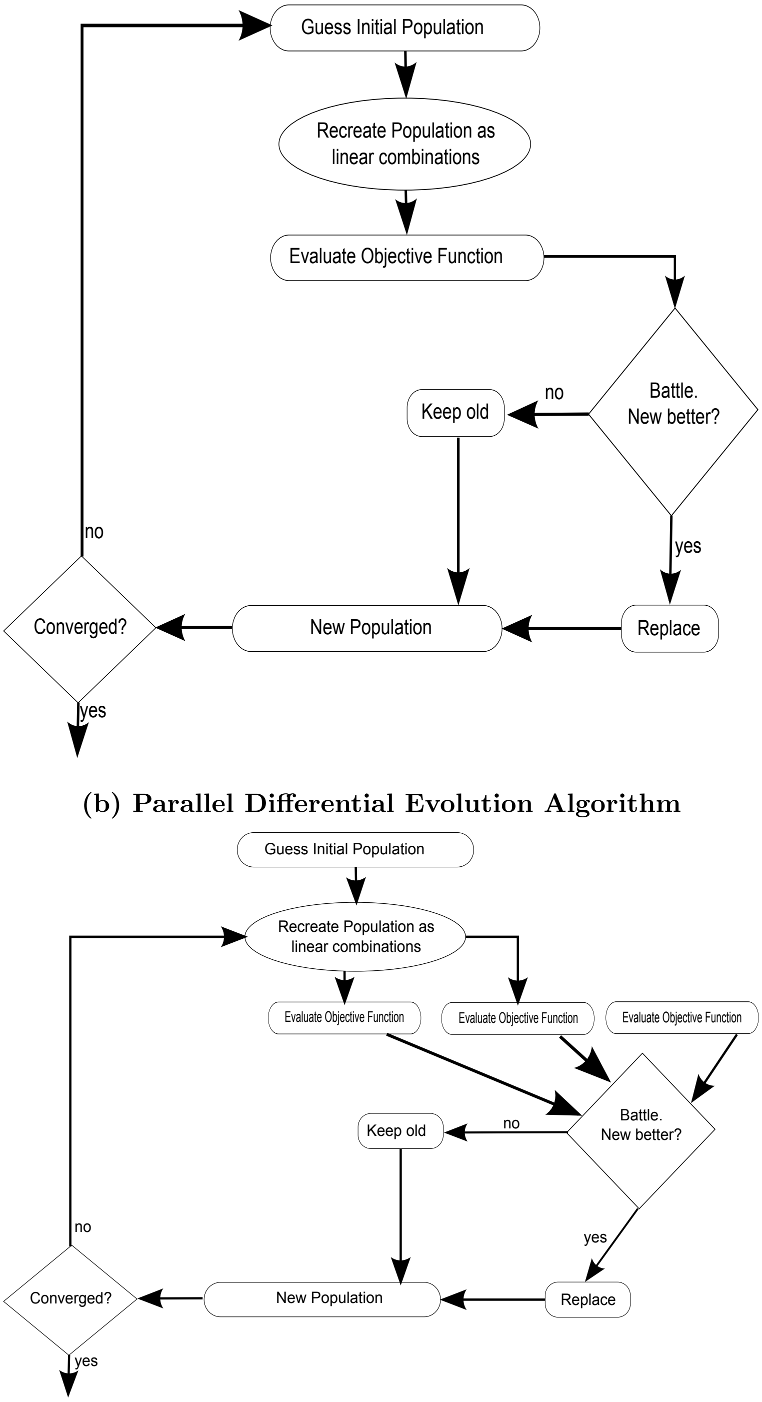
Parallelization could of course also be done at the model evaluation level. For example, at each time iteration the model usually has to be solved at a large number of states. The problem with this approach is that the results have to be aggregated much more often. This

[2]The open source software can be obtained from http://code.google.com/p/gc3pie/.

Figure IV.2: Differential evolution algorithm

(a) Differential Evolution Algorithm

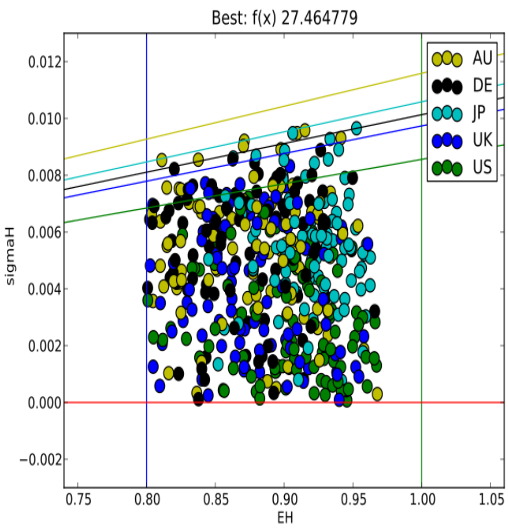
(b) Parallel Differential Evolution Algorithm



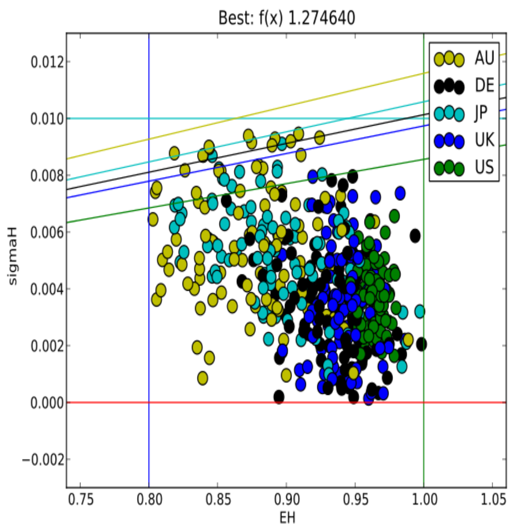
The top panel displays the differential evolution algorithm. The figure is based on Figure 1, page 3 in Chakraborty (2008). The bottom panel depicts how we parallelize the algorithm.

Figure IV.3: Evolution of parameter combinations

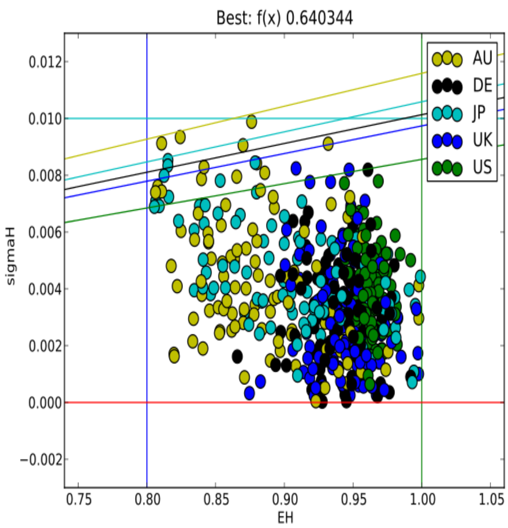
(a) Initial population



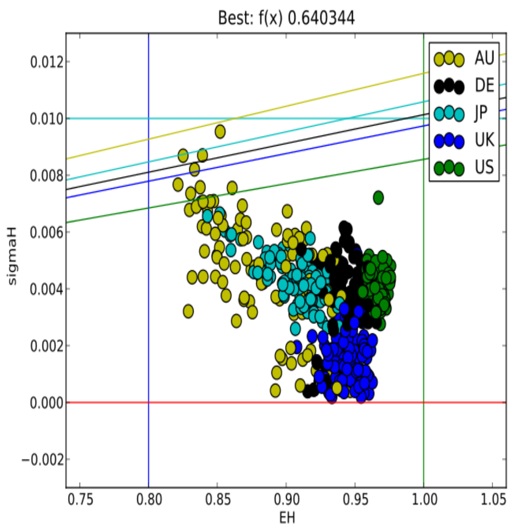
(b) Population after 40 iterations



(c) Population after 120 iterations



(d) Population after 240 iterations



The subfigures track the evolution of parameter guesses of the global optimization routine through iterations. Each point represents one parameter combination investigated. Countries are represented by different colors. Straight lines indicate constraints that the solver fulfills. Panel (a) depicts the initial population. Panel (b), (c), (d) show the population after 40, 120 and 240 iterations. The target value of the current best guess is reported above each subplot.

usually requires that all jobs run on one grid computer with shared memory which is much more expensive than a loose grid. Thus, the fact that the model needs to be evaluated at various parameter combinations can be exploited to reduce computation costs by placing the parallelization at this “highest” level in the analysis.

IV.5 Conclusion

This writeup provides an overview of different avenues to pick parameters in complex economic models. I summarize the calibration methodology and the controversial debate that followed its first use. Further I explore state-of-the art estimation techniques including likelihood and moment based techniques to estimate model parameters. Finally, I use a specific example of a complex model and show how today’s computer technology in connection with the ideas outlined in the beginning can be exploited to pin down model parameters.

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Part V

Asset Pricing with Idiosyncratic Risk: The Impact of Job Loss

Asset Pricing with Idiosyncratic Risk: The Impact of Job Loss

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Abstract

This paper studies the impact of unemployment risk on risk premia in an incomplete markets economy with many infinitely-lived heterogeneous agents. Job loss is modeled as large, but rare, persistent idiosyncratic shocks with heteroskedastic countercyclical volatility. Within an otherwise standard model and despite conservative assumptions on preferences, we simultaneously generate a sizeable equity premium and a low risk-free rate.

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V.1 Introduction

Mehra and Prescott (1985) show that the representative-agent complete markets model cannot replicate essential empirical facts in finance. An important strand of the literature has identified idiosyncratic risk as a potential explanation for the observed asset prices[1]. However, attempts to model idiosyncratic risk, generally involve modeling heterogeneous agents with incomplete markets. Since it is usually necessary to track one state variable per agent, such models quickly become intractable. One approach in the literature has been to reduce the state space by assuming the endogenous policy functions to be independent of each other.[2] This obviously remediates the problem of intractability, however, arguably also avoids the multidimensional nature of the problem.

In this paper we follow a different strand of literature[3] and employ the approximation algorithm of Smolyak (1963). It breaks the curse of dimensionality by picking interpolation points in a clever way. As a result, computing time grows only polynomially rather than exponentially as the number of state variables increases. This allows us to analyze more involved models of idiosyncratic risk with up to six agents without simplifying assumptions on policies. In particular, we are able to model job loss. By definition, idiosyncratic risk has to cancel out on the aggregate level. Thus, the loss of one agent must be the gain of the others. If one attempts to model unemployment in a two-agent economy then the employment income will be unrealistically large, which has severe implications on asset prices. Extending the analysis to a larger number of agents mitigates this problem and makes an analysis of a skewed income distribution feasible.

Within a Lucas (1978) framework we incorporate several model extensions suggested in the literature of idiosyncratic risk. Mankiw (1986) finds that the more concentrated shocks are on a small part of the population, the higher the risk premium. This is the case for job loss, which we model as large, but rare, idiosyncratic shocks. Among many others, Lucas (1994) and Heaton and Lucas (1996) stress the importance of market frictions. We assume that markets are dynamically incomplete: No asset allows direct insurance of income shocks and when unemployed, agents face a tight borrowing constraint. The constraint prevents them from smoothing consumption through borrowing in bad times and repaying in good times. Constantinides and Duffie (1996) rely on persistent idiosyncratic shocks with heteroskedastic countercyclical volatility. Empirically, unemployment is a lagged indicator of economic growth. Our model takes this co-movement into account by assuming that unemployment risk is high in recessions and low in booms and that agents remain unemployed until the economy picks up again.

The novel combination of these features generates realistic risk premia, despite low aggregate income growth volatility and conservative assumptions on preferences. A realistic calibration for the United States results in an equity premium of 4.7% with a risk-free rate

[1]See among many others Bewley (1982), Mankiw (1986), Weil (1992), Telmer (1993), Lucas (1994), Heaton and Lucas (1996) or Constantinides and Duffie (1996).

[2]See Aiyagari (1994), Krusell and Smith (1997), Krusell and Smith (1998), Storesletten, Telmer, and Yaron (2007).

[3]Krueger and Kubler (2004), Krueger and Kubler (2006) and Malin, Krueger, and Kubler (2011).

of 1.4%. Thus, the model generates large risk premia despite low risk-free rates.

The paper is organized as follows: Section 2 describes the model. Section 3 specifies the unemployment dynamics. The calibration is found in section 4, followed by the results in section 5. Finally, section 6 concludes.

V.2 The model

V.2.1 Economy

We consider an endowment economy, populated by n infinitely-lived agents. We denote average income per agent as Y_t and assume aggregate income (nY_t) to grow with a stochastic rate $g_{t+1} = \frac{Y_{t+1}}{Y_t}$.

V.2.2 Preferences

Each agent has the same recursive Epstein and Zin (1989) preferences over consumption C_t

$$V_t(W_t, z_t) = \left[(C_t - \varsigma Y_t)^{1-\rho} + \beta \mathbb{E}_t \left[V_{t+1}(W_{t+1}, z_{t+1})^{1-\gamma}\right]^{\frac{1-\rho}{1-\gamma}}\right]^{\frac{1}{1-\rho}}, \tag{V.1}$$

where $\psi = \frac{1}{\rho}$ is the intertemporal elasticity of substitution (IES), γ the risk aversion, β the subjective time discount factor and ς the relative subsistence level of consumption as a fraction of average income (Y_t). The subsistence level captures the idea that households need a minimum level of consumption to survive. More concretely, we think of the subsistence level as the level of consumption necessary to provide a household with basic needs as discussed in Sharif (1986). Utility is then derived only from consumption which exceeds the basic needs of survival. The state of the economy can be summarized by the wealth vector $W_t = (W_t^1, W_t^2, \ldots, W_t^n)$ and the current shock z_t. The current shock describes the aggregate state of the economy as well as the individual level of income.

V.2.3 Assets

One firm produces the entire output nY_t of the economy. The firm liquidates everything at the beginning of each period and splits the output into payoffs to employees ($Y_{l,t}$), bond holders ($P_{b,t}$) and stock holders ($P_{s,t}$)

$$Y_t = Y_{l,t} + P_{b,t}\bar{b} + P_{s,t}\bar{s},$$

where $\bar{b}$ and $\bar{s}$ denote the supply of bonds and stocks.

Typically, the claims of bond holders and employees are senior to the claims of stock holders. Abstracting from the possibility of default, we follow an extension in Mehra and Prescott (1985) and define stock payoffs as the stochastic part of the economy

$$P_{s,t}\bar{s} = Y_t - (1 - \bar{s})\mathbb{E}_{t-1}[Y_t].$$

Then aggregate wages and payouts to bond holders are fractions of the anticipated output of the economy

$$P_{b,t} = \mathbb{E}_{t-1}[Y_t],$$
$$Y_{l,t} = \bar{l}\mathbb{E}_{t-1}[Y_t],$$

where $\bar{l}$ is the share of wages as part of the expected aggregate firm output.

To acquire claims on output next period, people can invest in the firm at the end of each period by buying bonds (b_t^i) or investing in stocks (s_t^i). In total all claims need to equal the amount available to distribute

$$\sum_{i=1}^{n} b_t^i = \bar{b},$$
$$\sum_{i=1}^{n} s_t^i = \bar{s}.$$

V.2.4 Idiosyncratic risk

Similar to Lucas (1994), we introduce idiosyncratic shocks to agents' income Ψ_t^i. Then, agents' labour income can be decomposed into the non-stochastic part ($Y_{l,t}$) and an idiosyncratic shock: $Y_{l,t}^i = Y_{l,t} + \Psi_t^i$. To match aggregate income, the idiosyncratic shocks need to sum up to zero $\sum_{i=1}^{n} \Psi_t^i = 0$.

Then, the budget constraint is

$$W_t^i + Y_{l,t}^i = Q_{s,t}s_t^i + Q_{b,t}b_t^i + C_t^i \tag{V.2}$$

and wealth accumulates according to

$$W_{t+1}^i = P_{s,t+1}s_t^i + P_{b,t+1}b_t^i. \tag{V.3}$$

Furthermore, we assume that agents face state-contingent borrowing constraints. End of period net wealth of agent i (investment) has to lie above some minimum fraction of average income $\underline{i}_t^i < 0$

$$Q_{s,t}s_t^i + Q_{b,t}b_t^i \geq \underline{i}_t^i \bar{Y}_t.$$

V.3 Unemployment dynamics

The previous section described the general framework. In this section, we specify the structure of idiosyncratic shocks. While they are part of the model assumptions, we devote an entire section to unemployment dynamics for two reasons: First, to reflect the importance

Table V.1: Aggregate Markov chain

(a) Shocks

State	g_t
1	$\mu_g - \sigma_g$
2	$\mu_g + \sigma_g$

(b) Transition

State	1	2
1	p	$1-p$
2	$1-p$	p

idiosyncratic shocks play for the emerging model predictions. Second, to freely discuss computational considerations and impacts on calibration choices.

To make models numerically tractable, it is necessary to discretize the aggregate and individual shock space. The usual approach is to write the model in continuous states and then apply a discretization method, such as Tauchen and Hussey (1991). However, in this paper, we choose to model the dynamics of exogenous processes directly in a discrete shock space.

This leads to the usual trade-off between computational feasibility (a small number of states) and a realistic setting (a large number of states). The idea behind our Markov chain is to minimize the number of states, while maintaining the necessary components to model aggregate and idiosyncratic risk.

V.3.1 Markov chain

Similar to Mankiw (1986), our Markov chain is composed of two parts: Aggregate shocks indicate the distribution of average income over time. Idiosyncratic shocks specify the distribution of income across agents.

Aggregate Markov chain

Table V.1 shows the discretization of the aggregate economy's growth rate (g_t) into a Markov chain with two states. Economic growth is high in one state and low in the other. To match the first two unconditional moments, we construct the shock matrix by subtracting and adding the observed standard deviation to the observed mean. The corresponding transition matrix is parameterized by p, denoting the probability to remain in the same growth state. Empirically, income growth persistence is small (see Table V.5). Thus, for simplicity, we assume i.i.d income growth, i.e. $p = 0.5$.

Individual Markov chain

We model job loss through a separate Markov chain displayed in Table V.2. The shock matrix specifies idiosyncratic shocks and consists of n rows, where each row i represents the state in which agent i falls unemployed. Each agent's shock to income is represented by one column in the matrix. The entries display the percent deviation of agent i's labor income from average income, denoted $\psi_t^i = \frac{\Psi_t^i}{Y_{l,t}}$. Agent i suffers from job loss in state i and

Table V.2: Individual Markov chain

State	ψ_t^1	ψ_t^2	ψ_t^3	$\dots$	ψ_t^n
1	$-\Delta$	$\frac{\Delta}{n-1}$	$\frac{\Delta}{n-1}$	$\dots$	$\frac{\Delta}{n-1}$
2	$\frac{\Delta}{n-1}$	$-\Delta$	$\frac{\Delta}{n-1}$	$\dots$	$\frac{\Delta}{n-1}$
3	$\frac{\Delta}{n-1}$	$\frac{\Delta}{n-1}$	$-\Delta$	$\dots$	$\frac{\Delta}{n-1}$
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\ddots$	$\vdots$
n	$\frac{\Delta}{n-1}$	$\frac{\Delta}{n-1}$	$\frac{\Delta}{n-1}$	$\dots$	$-\Delta$

thus receives a lower income. Agent j receives a small positive income adjustment in state i to cancel out agent i's shock at the aggregate. For example, in state 1 agent 1 receives an income shock of $\psi_t^1 = -\Delta$. Thus agent 1's labor income amounts to $Y_{l,t}^1 = (1-\Delta)Y_{l,t}$. Similarly agent 2's income in state 1 is increased by $\frac{\Delta}{n-1}$ resulting in a labor income of $Y_{l,t}^2 = \left(1 + \frac{\Delta}{n-1}\right) Y_{l,t}$.

Common Markov chain

Finally, Table V.3 combines the individual and the aggregate into a common Markov chain. First, unemployment only occurs in an economic downturn. Second, once unemployed, an agent remains unemployed with probability p and regains employment when the economy recovers with probability $1-p$. Since job loss only occurs in an economic downturn, idiosyncratic risk is countercyclical and heteroskedastic.

This construction has two advantages: Economically, it incorporates countercyclical heteroscedastic idiosyncratic shocks. Computationally, the number of states is the number of agents plus one, thus as small as possible.

V.3.2 Intuition

The next two sections show that with the above structure of unemployment dynamics, the model is capable of simultaneously generating large risk premia and low risk-free rates with modest and conservative calibration choices. In this subsection we attempt to provide an intuition for this result.

Unemployment constitutes a large, potentially long lasting, hit on agents' income. From a partial equilibrium perspective, agents generally have three ways to smooth consumption as a response to idiosyncratic shocks. First, agents can buy direct ex-ante insurance. Second, agents can save as a precaution. Third, agents can indebt themselves, whenever income is low and pay it back later.

In our model, agents cannot buy ex-ante insurance against future job losses, since markets are dynamically incomplete. Indebting is limited through the borrowing constraint. Therefore, agents have to rely mainly on precautionary savings to smooth their consumption. Figure V.1 illustrates the development of wealth over the unemployment cycle. Initially, the agent will build up wealth as a precaution for a potential future job

Table V.3: Common Markov chain

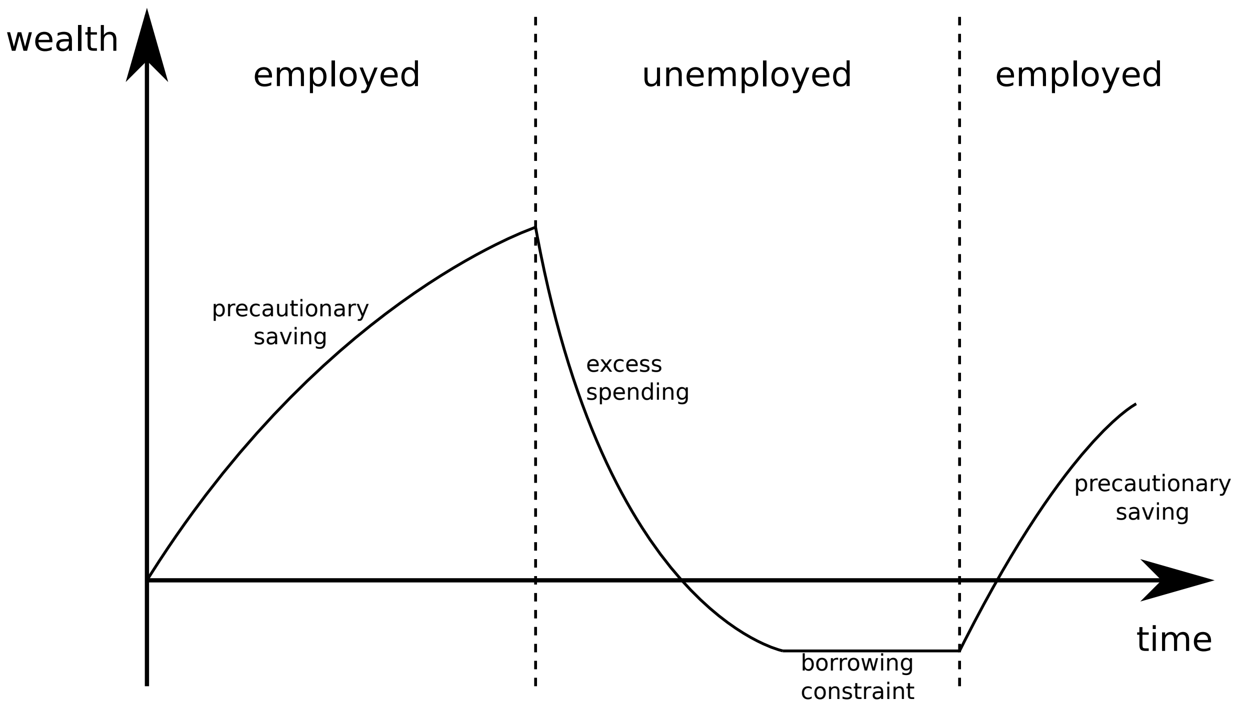
(a) Shocks

State	g_t	ψ_t^1	ψ_t^2	ψ_t^3	$\dots$	ψ_t^n
1	$\mu_g - \sigma_g$	$-\Delta$	$\frac{\Delta}{n-1}$	$\frac{\Delta}{n-1}$	$\dots$	$\frac{\Delta}{n-1}$
2	$\mu_g - \sigma_g$	$\frac{\Delta}{n-1}$	$-\Delta$	$\frac{\Delta}{n-1}$	$\dots$	$\frac{\Delta}{n-1}$
3	$\mu_g - \sigma_g$	$\frac{\Delta}{n-1}$	$\frac{\Delta}{n-1}$	$-\Delta$	$\dots$	$\frac{\Delta}{n-1}$
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\ddots$	$\vdots$
n	$\mu_g - \sigma_g$	$\frac{\Delta}{n-1}$	$\frac{\Delta}{n-1}$	$\frac{\Delta}{n-1}$	$\dots$	$-\Delta$
n+1	$\mu_g + \sigma_g$	0	0	0	$\dots$	0

(b) Transition

State	1	2	3	$\dots$	n	$n+1$
1	p	0	0	$\dots$	0	$1-p$
2	0	p	0	$\dots$	0	$1-p$
3	0	0	p	$\dots$	0	$1-p$
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\ddots$	$\vdots$	$\vdots$
n	0	0	0	$\dots$	p	$1-p$
$n+1$	$\frac{1-p}{n}$	$\frac{1-p}{n}$	$\frac{1-p}{n}$	$\dots$	$\frac{1-p}{n}$	p

Figure V.1: Illustration of wealth dynamics



loss. Upon job loss, the agent enters a phase of excess spending, consuming the accumulated wealth. After all savings are depleted, agents start borrowing. Eventually, however, the borrowing constraint will hit. From this point on, consumption is limited to income. When the economy picks up again and the agent regains employment, the cycle starts over from the beginning with precautionary saving.

All in all, precautionary savings is the primary way for agents to smooth idiosyncratic income shocks. High asset demand raises prices and reduces returns, allowing us to generate large risk-premia while maintaining a realistically low risk-free rate.

V.4 Calibration

This section discusses the model calibration. Table V.4 summarizes discretionary choices, while the top panel of Table V.5 shows estimated input data.

V.4.1 Population size

The economy is populated by six agents ($n = 6$). According to the specification of the unemployment dynamics six agents imply an average unemployment rate of 8.3%. As a comparison, the US post war average is 5.8%. Figure V.2 displays the historical evolution. One can see large fluctuations over time, ranging from below 3% to almost 10%. The empirical annual volatility is 1.6%. Our model implied annual volatility is somewhat larger with 4.2%.

V.4.2 Preferences

We distinguish two calibration choices in the second panel of Table V.4. The third column specifies the parameterization under constant relative risk aversion (CRRA) ($\psi = \frac{1}{\gamma}$). We set risk aversion (γ) to 5. We adapt the discount factor (β) to arrive at reasonable values for the risk-free rate. In particular we set $\beta = 0.97$.

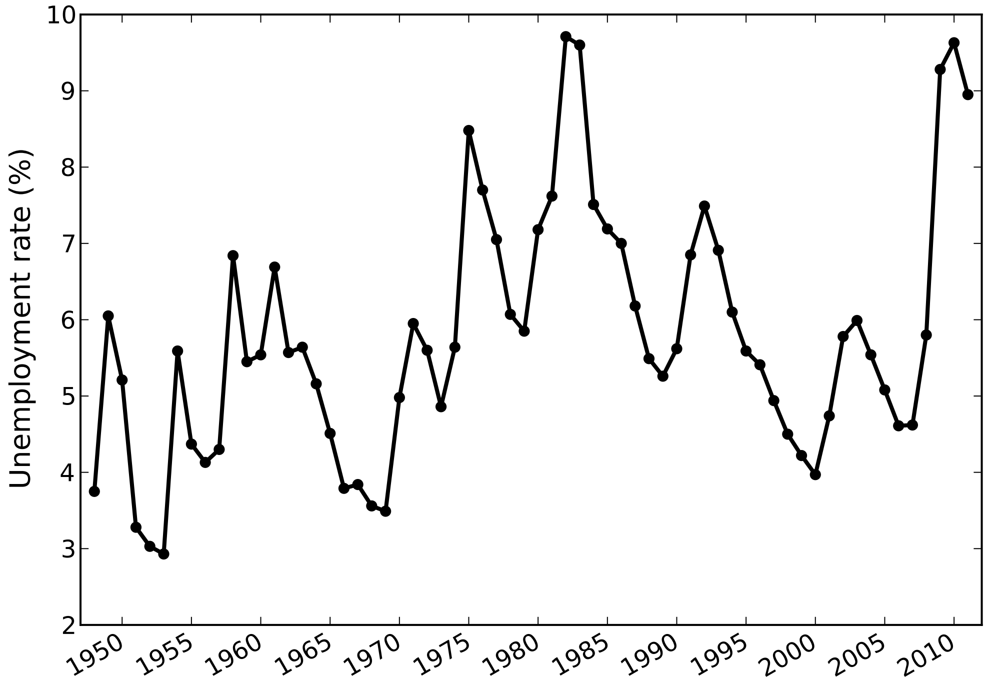
Since the risk premium under this specification is still relatively small, we investigate the effect of increasing γ (reducing ψ). Increasing risk aversion reduces ψ in the CRRA framework. Empirically ψ is estimated to be "significantly different from zero, and probably close to 1"[4]. Thus, as we increase risk aversion it seems plausible to move away from the standard CRRA preferences and adjust ψ. The fourth column describes the parameterization under the general Epstein-Zin preferences where we set risk aversion to 8 and the intertemporal elasticity of substitution to 0.33. Again, to arrive at a reasonable risk-free rate we adjust the discount factor, in this case 0.99.

We set the subsistence level (ς) in both parameterizations to 10%. Thus, we assume that the US median-income household of $\$49,400$[5] cannot survive with less than $\$5,000$.

[4]Beaudry and Wincoop (1996). Hansen and Singleton (1982) and Campbell and Cochrane (1999) also find the IES to be larger than $\frac{1}{\gamma}$.

[5]DeNavas-Walt, Proctor, and Smith (2011).

Figure V.2: Unemployment rate



Seasonally adjusted unemployment rate over time, 16 years and older, annual averages, provided by the U.S. Department of Labor.

Table V.4: Parameters

Parameter		CRRA	EZ
Number of agents	n	6	6
Discount factor	β	0.97	0.99
Risk aversion	γ	5	8
IES	ψ	0.20	0.33
Subsistence level	ς	10%	10%
Bond supply	$\bar{b}$	20%	20%
Stock supply	$\bar{s}$	15%	15%
Borrowing constraint, when unem.	$\underline{i}$	-5%	-5%
Replacement rate	$1-\Delta$	45%	45%

This table displays two calibration choices. The left column represents preferences with constant relative risk aversion (CRRA), $\gamma = 1/\psi$. The right column represents Epstein-Zin (EZ) preferences, $\gamma \neq 1/\psi$.

Table V.5: Moments — base calibration

Parameter		Data	CRRA	EZ
Avg. income growth	μ_g	3.3%	3.3%	3.3%
Income growth volatility	σ_g	2.8%	2.8%	2.8%
Income growth AC	$AC[g]$	−0.1%	0%	0%
Avg. consumption growth	μ_g	3.4%	3.3%	3.3%
Consumption growth vola.	σ_g	2.0%	2.8%	2.8%
Consumption growth AC	$AC[g]$	9.9%	0%	0%
Idiosyncratic income vola	$\sigma\left[Y_{t+1}^i/Y_t^i\right]$	25.1%	40.0%	40.0%
Idiosyncratic cons. vola	$\sigma\left[C_{t+1}^i/C_t^i\right]$	6% − 12%	14.4%	15.2%
Avg. market return	$\mathbb{E}[R^m]$	8.7%	5.1%	6.1%
Market return vola.	$\sigma[R^m]$	17.1%	18.9%	19.1%
Avg. risk-free rate	$\mathbb{E}[R^f]$	1.4%	1.6%	1.4%
Risk-free rate vola.	$\sigma[R^f]$	2.5%	2.4%	1.7%
Risk premium	$\mathbb{E}[R^m - R^f]$	7.3%	3.5%	4.7%

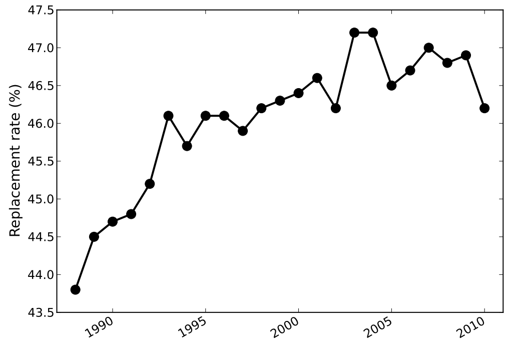
This table compares model implied moments with empirical observations. The two calibrations for CRRA and EZ preferences are displayed in the two rightmost columns. The data sources are as follows:
Income growth: BEA, Real Gross Domestic Product, seasonally adjusted, 1947-2010.
Consumption growth: OECD, private final consumption expenditure, 1947-2010.
Idiosyncratic income volatility: From Heaton and Lucas (1996).
Idiosyncratic consumption volatility: From Brav, Constantinides, and Geczy (2002).
Market return: Value weighted NYSE, including dividends, 1947-2010, deflated by CPI from CRSP.
Risk-free rate: T-Bills 90 days, deflated by CPI from CRSP.

V.4.3 Financial economy

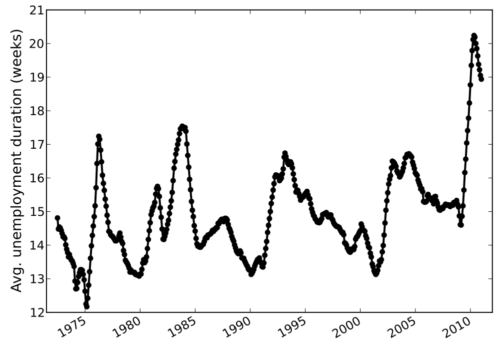
The third panel of Table V.4 displays the parameterization of the financial economy. The firm in our economy distributes its production through labor, bond and stocks. We assume these shares to be 65% for labor ($\bar{l}$), 20% for bonds ($\bar{b}$) and 15% for stocks ($\bar{s}$). We also enforce a borrowing constraint, denoted $\underline{i}$, in the unemployment state only. This is computationally easier and does not matter economically since agents in employment have no reason to borrow. In the real world unemployed agents have hard time borrowing beyond negative net wealth. In this situation credit cards tend to be one of the few ways to borrow but without a proper proof of employment limits tend to be tight. Heaton and Lucas (1996) argue a value between 0% and −10% is reasonable. We pick the middle and choose the boundary on net wealth of the unemployed ($\underline{i}$) as −5%.

Figure V.3: Unemployment

(a) Replacement rate



(b) Average benefit collection time



Replacement rate: "Average Weekly UI [unemployment insurance] Benefit as a Percent of Average Weekly Wage", US Department of Labor.
Average benefit collection time: "The average number of weeks for which unemployment insurance claimants collect benefits under regular state programs", US Department of Labor.

V.4.4 Real economy

Aggregate growth

The first two panels of Table V.5 depict aggregate income and consumption growth. Income growth refers to the real gross domestic product, seasonally adjusted from the U.S. Bureau of Economic Analysis over the horizon 1947 - 2010. Consumption is private final consumption expenditure, also seasonally adjusted over the same time horizon, obtained from the Organisation for Economic Co-operation and Development (OECD). Due to the lack of a savings technology, aggregate income is identical to aggregate consumption. Matching income growth as opposed to consumption growth is an arbitrary choice.

Unemployment

Calibrating unemployment requires choosing two parameters: first, the *replacement rate* $(1 - \Delta)$, how much income unemployed agents receive relative to how much they received in employment; second, the *duration of unemployment*, the average time it takes agents to regain employment.

Figure V.3a shows the replacement rate time series for the United States from 1988 to 2010. In 1988 the replacement rate was about 44% increasing to about 47% in the last decade. In a longer perspective, government transfers have increased over time. Therefore, historically the replacement rate has certainly been much lower. We take the conservative value of 45% as the replacement rate $(1-\Delta)$. Thus, falling unemployed implies an income drop of 55%.

Autocorrelation in US aggregate income growth is very small (see Table V.5). Thus, we model income growth as a random walk. As the model is computed in quarterly frequency, this implies, by construction of the Markov chain, an expected unemployment time of 6 months[6]. It is unclear which empirical proxy for average unemployment time to look at. One possibility is to look at average benefit collection time, see Figure V.3b. Historically, it has been between 13 and 20 weeks. However, as many people in unemployment remain unemployed past the stage of entitlement to benefits, the average duration of unemployment is certain to be higher. Thus, we believe, the implied value of 6 months, e.g. about 25 weeks is reasonable.

V.5 Results

In this section we first discuss the results under the main calibration. Next we discuss how the model results are affected by changes in the calibration and attempt to demonstrate the underlying mechanisms. For this purpose, we present three scenarios. First, we analyze the implications of Epstein-Zin preferences by varying risk aversion and IES. Second, we display comparative statics by presenting changes in a single input parameter. Third, we look at the impact of job loss by considering the alternative scenario of no idiosyncratic risk.

V.5.1 Base calibration

Table V.5 displays empirical moments obtained from simulating the model under the two base calibrations for CRRA and EZ discussed in the previous chapter. The second panel displays aggregate consumption. As we do not model a production side, aggregate consumption is identical to aggregate income. Therefore, while income is matched, consumption moments obviously deviate from the empirical ones.

The third panel shows idiosyncratic income and consumption volatility. The respective empirical moments are taken from the literature (Heaton and Lucas (1996) and Brav, Constantinides, and Geczy (2002)). While slightly larger, the model implied moments are reasonably close.

The fourth panel displays the main result of the paper: Averages and volatilities of market return and riskfree rate and the implied risk premium. In both cases the risk-free rate and volatilities are very close to the empirical values. For the CRRA case the model generates a risk premium of 3.5%, with the Epstein-Zin calibration the risk premium is 4.7%.

V.5.2 Epstein-Zin implications

We would like to understand in more detail the effects of Epstein-Zin preferences. For this purpose, Table V.6 displays different combinations of risk aversion and intertemporal

[6]Every 3 months an unemployed agent has a chance $p = 0.5$ to regain employment. Thus, the expected unemployment time is $\sum_{i=1}^{\infty} p^i 3i = 3\frac{p}{(1-p)^2} = 6$ months.

Table V.6: Asset prices with Epstein-Zin

γ	ψ	β	$\mathbb{E}[R^m]$	$\mathbb{E}[R^f]$	$\sigma[R^m]$	$\sigma[R^f]$	$\mathbb{E}[R^m - R^f]$
5	0.20	0.97	5.1%	1.6%	18.8%	2.4%	3.5%
5	0.33	0.97	9.9%	6.6%	20.1%	1.2%	3.3%
8	0.20	0.97	−1.9%	−6.5%	17.1%	3.2%	4.6%
8	0.33	0.97	7.2%	2.4%	19.1%	1.9%	4.8%
5	0.20	0.97	5.1%	1.6%	18.8%	2.4%	3.5%
5	0.33	1.04	4.3%	1.1%	19.2%	0.9%	3.1%
8	0.20	0.86	4.7%	−0.2%	17.8%	4.3%	4.8%
8	0.33	0.99	6.1%	1.4%	19.1%	1.7%	4.7%
	Data		8.7%	1.4%	17.1%	2.5%	7.3%

This table shows asset pricing moments for varying combinations of risk aversion (γ), intertemporal elasticity of substitution (ψ) and the discount factor (β). The first panel keeps the discount rate constant. The second panel varies the discount rate to obtain reasonable values for the risk-free rate. The last panel repeats the empirical observation from Table V.5.

elasticity of substitution. We distinguish two cases. In the upper panel we keep the discount rate constant at $\beta = 0.97$. In the lower panel, we adjust the discount rate to keep an almost constant risk-free rate. Let us first look at the upper panel with a constant discount rate. An increase in ψ implies a larger tolerance of different consumption levels across time — agents will smooth their consumption less, therefore the risk-free rate volatility falls. Furthermore, when we increase ψ, agents have less incentives to save as a precaution — demand for bonds and stock falls and interest rates as well as the market return rise. As ψ determines intertemporal choices, the impact on the risk premium is rather small. A change in γ increases volatilities and risk premium, since agents are more afraid of risk. An increased risk aversion also decreases the risk-free rate. Agents will have a greater fear of unemployment. They rely on precautionary savings to prevent losses and therefore asset prices rise, e.g. returns fall.

In the lower panel, we keep the risk-free rate almost constant to separate the effects more clearly. ψ has strong effects on the risk-free rate volatility, yet only minor indirect effects on the risk premium. An increase in risk aversion increases volatilities and the risk premium.

V.5.3 Sensitivities in preference parameters

Table V.7 shows how the model responds to changes in one of the input parameters. The first two lines repeat moments of the CRRA column of Table V.5 as a reference. In the following lines, we vary one input parameter from Table V.4 at a time. The value in parentheses repeats the respective value of the base case. We will now discuss the effects one by one.

Table V.7: Preferences — sensitivities

Case	$\mathbb{E}[R^m]$	$\mathbb{E}[R^f]$	$\sigma[R^m]$	$\sigma[R^f]$	$\mathbb{E}[R^m - R^f]$
Base	5.1%	1.6%	18.8%	2.4%	3.5%
β = 0.96 (0.97)	6.0%	2.5%	19.0%	2.4%	3.5%
β = 0.98 (0.97)	4.2%	0.8%	18.7%	2.3%	3.4%
γ = 4 (5)	9.5%	6.6%	19.9%	1.6%	2.9%
γ = 6 (5)	−0.7%	−4.6%	17.6%	3.0%	3.8%
ψ = 0.15 (0.2)	1.2%	−2.3%	18.2%	2.8%	3.5%
ψ = 0.25 (0.2)	7.8%	4.4%	19.4%	1.9%	3.4%
ς = 0 (10%)	11.3%	7.8%	20.2%	1.9%	3.5%
$\bar{s}$ = 10% (15%)	5.5%	0.4%	28.7%	3.2%	5.0%
$\bar{b}$ = 10% (20%)	1.6%	−1.8%	17.2%	3.7%	3.4%

The first line repeats the model results for the CRRA case from Table V.5. The following lines show deviations in one parameter. The first column describes the parameters, in parentheses we repeat the respective value in the base case. The following columns display the different moments for each calibration. β discount factor; γ risk aversion; ψ intertemporal elasticity of substitution; ς subsistence level; $\bar{s}$ stock supply; $\bar{b}$ bond supply.

The discount rate (β) works as expected. A lower discount rate implies a higher interest rate and vice versa. The effects on volatilities and the risk premium are negligible. The risk aversion (γ) and IES (ψ) show similar qualitative results as in Table V.6. Risk aversion decreases returns and increases risk premia. The IES affects primarily the returns and has only minor effects on risk premia. The subsistence level (ς) has a strong impact on asset returns. Agents are forced to a minimum level of consumption, thus unemployment is particularly painful. As a precaution, agents invest more in assets, e.g. asset prices rise and returns fall.

Reducing the stock supply ($\bar{s}$) implies an increase in the stock volatility. In our model, all aggregate risk is carried by stock holders. As there are less stocks, the relative risk increases. Therefore, the stock volatility increases to 28.7% and consequently the risk premium increases to 5.0%. The second effect is common for stock and bond supply ($\bar{b}$). Less supply, implies less possibilities to save, e.g. less supply of assets in general. As the supply of assets decreases, the price increases, e.g. asset returns fall.

V.5.4 Impact of job loss

Table V.8 shows the impact of the introduction of job loss. In the upper panel, we show possible model calibrations for the discount rate and risk aversion without idiosyncratic risk. Since, we calibrated aggregate risk on income rather than consumption, introduced a subsistence level and leveraged our firm, the model is capable of creating sizable risk premia even without idiosyncratic risk. However, at the cost of extremely large risk-free rates. With this result, we are in line with Mehra and Prescott (1985) and Weil (1989),

Table V.8: Impact of job loss

$1-\Delta$	γ	β	$\mathbb{E}[R^m]$	$\mathbb{E}[R^f]$	$\sigma[R^m]$	$\sigma[R^f]$	$\mathbb{E}[R^m - R^f]$
100%	2	0.90	24.5%	22.9%	24.0%	1.1%	1.6%
100%	2	0.99	13.2%	11.8%	21.9%	1.0%	1.4%
100%	5	0.90	46.1%	41.6%	28.7%	1.8%	4.5%
100%	5	0.99	33.0%	28.8%	26.1%	1.6%	4.1%
45%	2	0.90	19.4%	17.8%	22.4%	0.3%	1.6%
45%	2	0.99	8.8%	7.4%	20.4%	0.3%	1.4%
45%	5	0.90	11.6%	7.9%	19.9%	2.8%	3.7%
45%	5	0.99	3.4%	0.0%	18.6%	2.3%	3.4%
	Data		8.7%	1.4%	17.1%	2.5%	7.3%

This table compares different replacement rates $(1-\Delta)$. In the upper panel, the replacement rate is 100%, e.g. job loss has no effect. In the lower panel, the replacement rate is 45%, e.g. in case of unemployment, income drops by 55%. The lines report moments of asset prices for different values of risk aversion (γ) and (β) for preferences with constant relative risk aversion. The last line repeats the empirical observations from Table V.5 as a reference.

who show that when β is constrained to lie below 1, it is impossible to generate large risk premia, while keeping interest rates at a reasonable level. In this sense, we find the puzzle to be more about the historically low interest rate, rather than the equity premium.

The introduction of idiosyncratic risk in the lower panel opens the path for large risk premia and realistic risk-free rates. Since agents fear unemployment, they have strong incentives to save as a precaution. Therefore, asset prices rise and returns decrease to reasonable values. By calibrating $\gamma = 5$ and $\beta = 0.99$, we obtain a zero interest rate and still maintain a sizable risk premium. Therefore, our model results suggest, that the large risk premia are derived from aggregate risk, however, individual risks and the induced precautionary savings justify the historically low interest rate and answer the question raised by Weil (1989) "why is the risk-free rate so low?"

V.6 Conclusion

This paper relates risk premia to unemployment risk. Without idiosyncratic risk, we find it possible to generate large risk premia, but only at the cost of an unreasonably large interest rate. Similar to the literature[7], we thus conclude the equity premium puzzle to be more about the question why risk-free rates have been so low, rather than why equity returns have been so high. Unemployment risk provides an answer. When unemployed households face tight credit constraints and incomplete markets prevent them from insuring their idiosyncratic risk, they have to rely on precautionary savings to dampen the effects

[7] See Kocherlakota (1996) for a survey.

of unemployment. This creates strong demand for bonds and causes interest rates to fall to realistic levels.

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V.A Normalization

The optimization problem is the same for each agent. Thus, for notational convenience, we drop the agent specific superscript i in this section. Agents maximize utility (eq. (V.1) on page 92) with respect to the budget constraint (eq. (V.2) on page 93) and wealth accumulation (eq. (V.3) on page 93), i.e.

$$\max_{C_t, s_t, b_t} V_t(W_t, z_t) = \left[(C_t - \varsigma Y_t)^{1-\rho} + \beta \mathbb{E}_t \left[V_{t+1}(W_{t+1}, z_{t+1})^{1-\gamma} \right]^{\frac{1-\rho}{1-\gamma}} \right]^{\frac{1}{1-\rho}}, \tag{V.4}$$

s.t.

$$W_t + Y_{l,t} = Q_{s,t} s_t + Q_{b,t} b_t + C_t,$$

$$W_{t+1} = P_{s,t+1} s_t + P_{b,t+1} b_t.$$

To apply dynamic programming (Bellman (1957), Stokey and Lucas (1989)), we need our model to be stationary. To achieve this, we normalize all equations with the trending variable average income (Y_t). We denote the normalized variables in our model with lower case letters, i.e.

$$y_t = \frac{Y_t}{Y_t},\ v_t = \frac{V_t}{Y_t},\ v_{t+1} = \frac{V_{t+1}}{Y_{t+1}},\ w_t = \frac{W_t}{Y_t},\ q_{s,t} = \frac{Q_{s,t}}{Y_t},\ q_{b,t} = \frac{Q_{b,t}}{Y_t},\ c_t = \frac{C_t}{Y_t}.$$

The normalized payoffs to employees stock holders and bond holders are then

$$\begin{aligned}
y_{l,t} &= \frac{Y_{l,t}}{Y_t} = \frac{(1+\psi_t)\bar{l}\mathbb{E}_{t-1}[Y_t]}{Y_t} = \bar{l}(1+\psi_t)\frac{\mathbb{E}_{t-1}[g_t]}{g_t}, \\
p_{b,t} &= \frac{P_{b,t}}{Y_t} = \frac{\mathbb{E}_{t-1}[Y_t]}{Y_t} = \frac{\mathbb{E}_{t-1}[g_t]}{g_t}, \\
p_{s,t} &= \frac{P_{s,t}}{Y_t} = \frac{Y_t - (1-\bar{s})\mathbb{E}_{t-1}[Y_t]}{\bar{s} Y_t} = \frac{1 - (1-\bar{s})\frac{\mathbb{E}_{t-1}[g_t]}{g_t}}{\bar{s}}.
\end{aligned}$$

We arrive at the normalized optimization problem by dividing (V.4) through Y_t. The normalized value function v_t has the additional factor g_{t+1} adjusting tomorrow's value to account for economic growth (see (V.5)). In the normalized version of our model, we choose to normalize prices and payoffs instead of asset holdings (see (V.6) and (V.7))

$$\max_{c_t, s_t, b_t} v_t(w_t, z_t) = \left[(c_t - \varsigma)^{1-\rho} + \beta \mathbb{E}_t \left[(g_{t+1} v_{t+1}(w_{t+1}, z_{t+1}))^{1-\gamma} \right]^{\frac{1-\rho}{1-\gamma}} \right]^{\frac{1}{1-\rho}}, \tag{V.5}$$

s.t.

$$w_t + y_t = q_{s,t} s_t + q_{b,t} b_t + c_t, \tag{V.6}$$

$$w_{t+1} = p_{s,t+1} s_t + p_{b,t+1} b_t. \tag{V.7}$$

V.B Equilibrium conditions

An analytic solution to (V.5) subject to (V.6) and (V.7) is unknown. To find a numeric solution, we solve the first order conditions using a nonlinear equation solver. In this section we derive the first order conditions starting from the Lagrangian.

V.B.1 Lagrangian

The Lagrangian for the normalized problem can be written as

$$\mathcal{L} = \left[(c_t - \varsigma)^{1-\rho} + \beta \mathbb{E}_t \left[g_{t+1}^{1-\gamma} v_{t+1}^{1-\gamma} \right]^{\frac{1-\rho}{1-\gamma}} \right]^{\frac{1}{1-\rho}} + \lambda \left[w_t + y_t - q_{s,t} s_t - q_{b,t} b_t - c_t \right]. \tag{V.8}$$

Note that we state the budget equation explicitly while substituting the wealth accumulation equation whenever necessary.

V.B.2 Derivatives

Differentiating (V.8) with respect to c_t and s_t yields

$$\begin{aligned} \frac{\partial \mathcal{L}}{\partial c_t} &= \frac{1}{1-\rho} \left[(c_t - \varsigma)^{1-\rho} + \beta \mathbb{E}_t \left[g_{t+1}^{1-\gamma} v_{t+1}^{1-\gamma} \right]^{\frac{1-\rho}{1-\gamma}} \right]^{\frac{1}{1-\rho}-1} (1-\rho)(c_t - \varsigma)^{-\rho} - \lambda = 0, \\ &= v_t^{\rho} (c_t - \varsigma)^{-\rho} - \lambda = 0, \end{aligned} \tag{V.9}$$

$$\frac{\partial \mathcal{L}}{\partial s_t} = \frac{1}{1-\rho} v_t^{\rho} \beta \frac{1-\rho}{1-\gamma} \left(\mathbb{E}_t \left[g_{t+1}^{1-\gamma} v_{t+1}^{1-\gamma} \right] \right)^{\frac{1-\rho}{1-\gamma}-1} \mathbb{E}_t \left[g_{t+1}^{1-\gamma} (1-\gamma) v_{t+1}^{-\gamma} \frac{\partial v_{t+1}}{\partial c_{t+1}} \frac{\partial c_{t+1}}{\partial s_t} \right] - \lambda q_{s,t} = 0.$$

From (V.9) we see that $\frac{\partial v_{t+1}}{\partial c_{t+1}} = v_{t+1}^{\rho} (c_{t+1} - \varsigma)^{-\rho}$. The one-period lagged eq. (V.6) together with the wealth accumulation eq. (V.7) imply $\frac{\partial c_{t+1}}{\partial s_t} = p_{s,t+1}$. Then the derivative with respect to stock holdings can be written as

$$\begin{aligned} \frac{\partial \mathcal{L}}{\partial s_t} &= v_t^{\rho} \beta \left(\mathbb{E}_t \left[g_{t+1}^{1-\gamma} v_{t+1}^{1-\gamma} \right] \right)^{\frac{\gamma-\rho}{1-\gamma}} \mathbb{E}_t \left[g_{t+1}^{1-\gamma} v_{t+1}^{-\gamma} v_{t+1}^{\rho} (c_{t+1} - \varsigma)^{-\rho} p_{s,t+1} \right] - \lambda q_{s,t} = 0, \\ &= v_t^{\rho} \beta \left(\mathbb{E}_t \left[g_{t+1}^{1-\gamma} v_{t+1}^{1-\gamma} \right] \right)^{\frac{\gamma-\rho}{1-\gamma}} \mathbb{E}_t \left[g_{t+1}^{1-\gamma} v_{t+1}^{\rho-\gamma} (c_{t+1} - \varsigma)^{-\rho} p_{s,t+1} \right] - \lambda q_{s,t} = 0. \end{aligned}$$

Differentiating with respect to bond holdings b_t yields

$$\frac{\partial \mathcal{L}}{\partial b_t} = v_t^{\rho} \beta \left(\mathbb{E}_t \left[g_{t+1}^{1-\gamma} v_{t+1}^{1-\gamma} \right] \right)^{\frac{\gamma-\rho}{1-\gamma}} \mathbb{E}_t \left[g_{t+1}^{1-\gamma} v_{t+1}^{\rho-\gamma} (c_{t+1} - \varsigma)^{-\rho} p_{b,t+1} \right] - \lambda q_{b,t} = 0.$$

V.B.3 Normalized first order conditions

Finally, we simplify the first order conditions normalizing by v_t^ρ. For this purpose define $\tilde{\lambda} = \frac{\lambda}{v_t^\rho}$. The first order conditions for consumption, stock holdings and bond holdings become

$$
\begin{aligned}
&(c_t - \varsigma)^{-\rho} - \tilde{\lambda} = 0,\\
&\beta \left(\mathbb{E}_t \left[g_{t+1}^{1-\gamma} v_{t+1}^{1-\gamma}\right]\right)^{\frac{\gamma-\rho}{1-\gamma}} \mathbb{E}_t \left[g_{t+1}^{1-\gamma} v_{t+1}^{\rho-\gamma} (c_{t+1} - \varsigma)^{-\rho} p_{s,t+1}\right] - \tilde{\lambda} q_{s,t} = 0,\\
&\beta \left(\mathbb{E}_t \left[g_{t+1}^{1-\gamma} v_{t+1}^{1-\gamma}\right]\right)^{\frac{\gamma-\rho}{1-\gamma}} \mathbb{E}_t \left[g_{t+1}^{1-\gamma} v_{t+1}^{\rho-\gamma} (c_{t+1} - \varsigma)^{-\rho} p_{b,t+1}\right] - \tilde{\lambda} q_{b,t} = 0.
\end{aligned}
$$

In the case of CRRA utility, $\rho = \gamma$. In this case, the first order conditions collapse to

$$
\begin{aligned}
&(c_t - \varsigma)^{-\gamma} - \tilde{\lambda} = 0,\\
&\beta \mathbb{E}_t \left[g_{t+1}^{1-\gamma} (c_{t+1} - \varsigma)^{-\gamma} p_{s,t+1}\right] - \tilde{\lambda} q_{s,t} = 0,\\
&\beta \mathbb{E}_t \left[g_{t+1}^{1-\gamma} (c_{t+1} - \varsigma)^{-\gamma} p_{b,t+1}\right] - \tilde{\lambda} q_{b,t} = 0.
\end{aligned}
$$

The full set of equilibrium conditions combines the above first order conditions with the market clearing conditions

$$
\begin{aligned}
\sum_{i=1}^{n} s_t^i &= \bar{s}, \quad \forall t.\\
\sum_{i=1}^{n} b_t^i &= \bar{b}, \quad \forall t.
\end{aligned}
$$

V.C Computation

V.C.1 State space

The state of the economy is described by beginning of period normalized wealth w_t of all agents and the current shock z_t.

V.C.2 Smolyak

Solving for all agents but one requires to find a full solution of the model, since all agents have identical preferences. The remaining agent simply receives all residual quantities. Thus, the state space dimension is equal to the number of agents minus one. The main challenge in solving the model is the approximation of policy functions as they depend on the entire wealth vector w_t. We compute the case of six agents, thus the dimension of the continuous state space is five. Consider, for example, a course grid of five points per axis. In this case, the number of grid points already amounts to $5^5 = 3125$. The algorithm proposed in Smolyak (1963) has the advantage that the number of points grows polynomially rather than exponentially in the number of dimensions, providing a counterspell to the curse of dimensionality. Applications of the Smolyak approximation algorithm in the field of economics first appeared in Krueger and Kubler (2004). Recent applications are Krueger and Kubler (2006) in an overlapping-generations model and Malin, Krueger, and Kubler (2011) in a multi-country real business cycle model. To our knowledge, this is the first time, Smolyak approximation is applied to an infinite horizon competitive equilibrium model.

V.C.3 Implementation of the borrowing constraint

An intuitive implementation of the borrowing constraint $I_t \geq \underline{I}$ through Kuhn-Tucker conditions leads to kinks in the policy function. The Smolyak algorithm, however, requires the approximated function to be smooth. To avoid kinks, we compute an auxiliary problem.

Consider the normalized wealth space $[\underline{w}, \overline{w}]$ for each agent[8]. First, we compute next period's policy over the state space ignoring the borrowing constraint. To account for the constraint, we then overwrite the optimal policy with a constant outside of the bounds. This ensures the borrowing constraints lie exactly at the boundary of the wealth space.

The above procedure may be equivalently formulated by the constraint, $I_t \geq I(s_t, \underline{w}, w)$, where $I(.)$ is the investment policy and w refers to the wealth vector of all other agents. Economically, this constraint is cumbersome and has no straight forward interpretation. However, equivalence to the original problem can be achieved by choosing $\underline{I} = I(s_t^u, \underline{w}, w)$ in the converged policy, where $w = \frac{\underline{w}+\overline{w}}{2}$ and s_t^u the unemployed state. Assuming that the borrowing constraint is non-binding in any state, in which the agent is employed and that the agents investment policy does not depend on the others wealth vector, the auxiliary and original problem yield the same solution.

To enforce the desired constraint $(\underline{i})$ on normalized investment (i_t) reported in the paper, we employ a (costly) optimization technique to arrive at according grid lower bound $(\underline{w})$. We solve the model on average about seven times until we arrive at the desired lower bound.

V.C.4 Policy iteration

We apply standard dynamic programming techniques as described in Judd (1998) to solve the model. As policies we choose to approximate the investment decision of each agent as a function of the state variables, specifying the exogenous shock and the wealth of each agent. Then we implement almost roll-over as the initial investment policy and initialise the value function for each agent as well as price policies for both assets, accordingly. Given these policies tomorrow, we solve for the optimal policies today at each Smolyak grid point

[8] This holds true for all agents except one. As described above, Smolyak requires a cubical state space. Thus, one agent's wealth space equalizes all others. In other words, the wealth space of one agent is the residual of all others, $[-(n-1)\overline{w}, -(n-1)\underline{w}]$. Economically this can be interpreted as one agent being capable to borrow almost infinite amounts.

and finally use the Smolyak algorithm to approximate the policies. We iterate backwards over time and repeat this procedure until investment policies, value functions and price policies are converged.

Finally, we simulate one million quarters and compute the empirical moments on individual consumption and asset returns reported in the paper.

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